Kenmore-Town of Tonawanda UFSD

We educate, prepare, and inspire all students to achieve their highest potential

Grade 4 Module 5 Parent Handbook

Fraction Equivalence, Ordering, and Operations

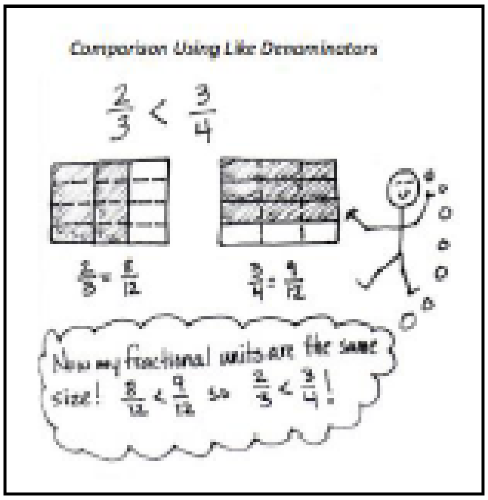
The materials contained within this packet have been taken from the Great Minds curriculum Eureka Math.

Eureka Math Tips for Parents

Grade 4
Module 5

Fraction Equivalence, Ordering, and Operations

In this 41-lesson module, students explore fraction equivalence and extend this understanding to mixed numbers. They compare and represent fractions and mixed numbers using a variety of models. Toward the end of the module, they use what they know to be true about whole number operations to apply to fractions and mixed number operations.



New Terms in this Module:

Benchmark Fraction - a known reference fraction by which other fractions can be measured, e.g. 0, ½, ¼, ¾, 1

Common denominator - when two or more fractions have the same denominator

Denominator - bottom number in a fraction

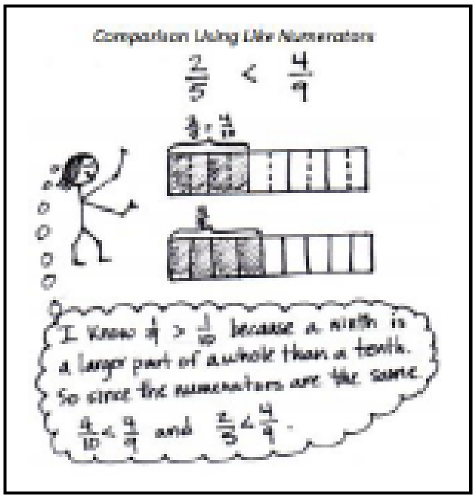
Line plot - display of data on a number line, using an x or another mark to show frequency

Mixed number - number made up of a whole number and a fraction

Numerator - top number in a fraction

Familiar Terms:

- *Compose*
- *Decompose*
- *Equivalent fractions*
- *Fractional unit*
- *Unit fraction*
- *Non-unit fraction*
- =, <, >



What Came Before this Module:
Students were introduced to many new geometrical terms and the relationships between them. They also learned to compose and classify two-dimensional figures.

What Comes After this Module:
In Module 6, students will use the understanding of fractions developed throughout Module 5, apply the same reasoning to decimal numbers, and build a solid foundation for later work with decimal operations.

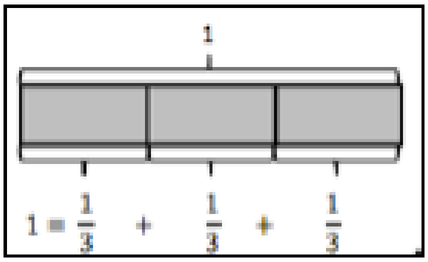
How you can help at home:

- Continue to practice and review multiplication and division math facts - this greatly supports work with fractions!
- Look for opportunities in daily life to discuss fractional parts and divide objects into equal parts.

Key Common Core Standards:

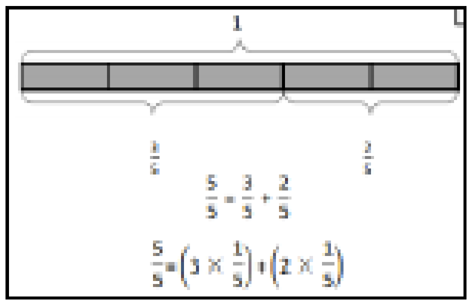
- *Generate and analyze patterns*
  - Generate a number or shape pattern that follows a given rule
- *Extend understanding of fraction equivalence and ordering*
  - Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models
  - Compare two fractions with different numerators and different denominators
- *Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers*
  - Understand a fraction a/b with a > 1 as a sum of fractions 1/b, e.g. 3/5 = 1/5 + 1/5 + 1/5
  - Apply and extend previous understandings of multiplication to multiply a fraction by a whole number
- *Represent and interpret data*
  - Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8)

Prepared by Erin Schweng, Math Coach



The tape diagram below shows how to break one whole into fifths, and then how those fifths can be grouped and added together to create the whole.

The tape diagram above shows a simple fraction addition problem in which each part of the tape is equal to one-third of the whole.



Spotlight on Math Models:

Tape Diagrams

You will often see this mathematical representation in *A Story of Units.*

A Story of Units has several key mathematical "models" that will be used throughout a student's elementary years.

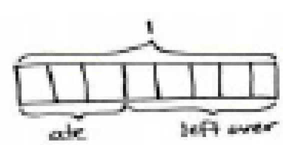
The tape diagram is a powerful model that students can use to solve various kinds of problems. Beginning in first grade, tape diagrams are used as simple models of addition and subtraction. Now in this fourth grade module, we will use them to model operations on fractions as well.

Tape diagrams are also called "bar models" and consist of a simple bar drawing that students make and adjust to fit a word or computation problem. They then use the drawing to discuss and solve the problem.

As students move through the grades, tape diagrams provide an essential bridge to algebra and solving for an unknown quantity. They are flexible mathematical tools that grow to fit students' needs as elementary mathematics increases in complexity.

Sample Problem from Module 5:
(Example taken from Lesson 19; note the use of a tape diagram to solve the problem)

Mr. Salazar cut his son's birthday cake into equal pieces. Mr. Salazar, Mrs. Salazar, and the birthday boy each ate 1 piece of cake. What fraction of the cake was left?



$\frac{5}{8}$ of the cake is left.

Solution 1

$1 - \frac{3}{8} = \frac{8}{8} - \frac{3}{8} = \frac{5}{8}$

Solution 2

$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + x = \frac{8}{8}$

$\frac{3}{8} + x = \frac{8}{8}$

$\frac{3}{8} + \frac{5}{8} = \frac{8}{8} \qquad x = \frac{5}{8}$

Fraction Equivalence, Ordering, and Operations

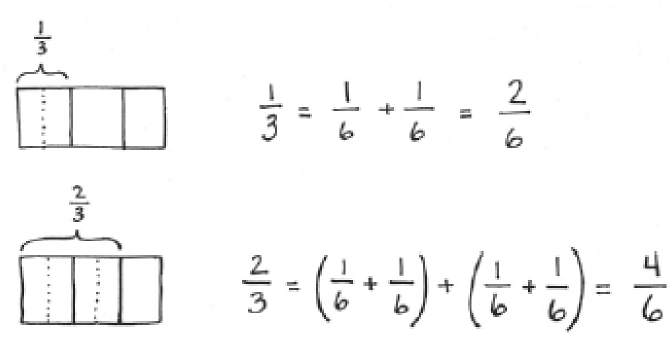
OVERVIEW

In this 45-day module, students build on their Grade 3 work with unit fractions as they explore fraction equivalence and extend this understanding to mixed numbers. This leads to the comparison of fractions and mixed numbers and the representation of both in a variety of models. Benchmark fractions play an important part in students' ability to generalize and reason about relative fraction and mixed number sizes. Students then have the opportunity to apply what they know to be true for whole number operations to the new concepts of fraction and mixed number operations.

Students begin Topic A by decomposing fractions and creating tape diagrams to represent them as sums of fractions with the same denominator in different ways (e.g., $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1}{5} + \frac{2}{5}$) (**4.NF.3b**). They proceed to see that representing a fraction as the repeated addition of a unit fraction is the same as multiplying that unit fraction by a whole number. This is already a familiar fact in other contexts.

For example, just as 3 twos = 2 + 2 + 2 = 3 × 2, so does 3 fourths = $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$.

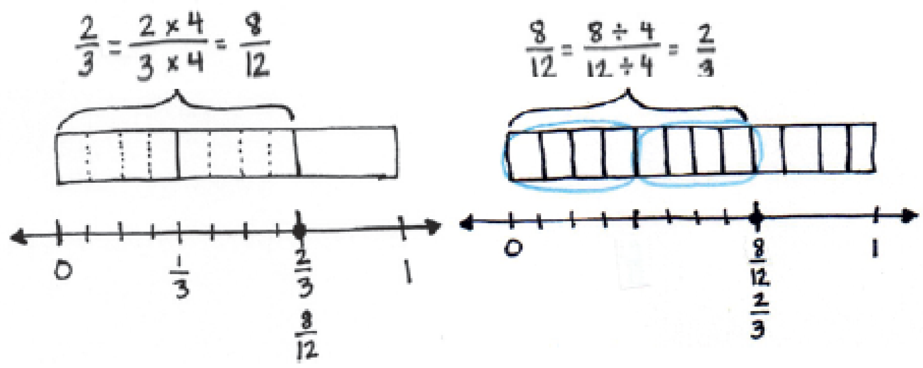
The introduction of multiplication as a record of the decomposition of a fraction (**4.NF.4a**) early in the module allows students to become familiar with the notation before they work with more complex problems. As students continue working with decomposition, they represent familiar unit fractions as the sum of smaller unit fractions. A folded paper activity allows them to see that, when the number of fractional parts in a whole increases, the size of the parts decreases.



They proceed to investigate this concept with the use of tape diagrams and area models. Reasoning enables them to explain why two different fractions can represent the same portion of a whole (**4.NF.1**).

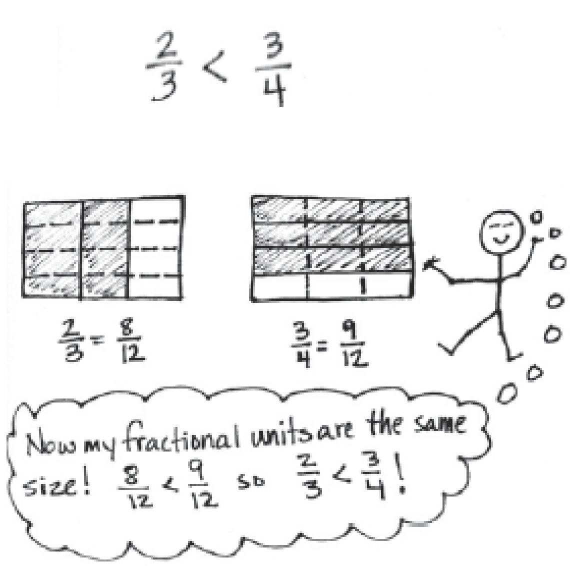
In Topic B, students use tape diagrams and area models to analyze their work from earlier in the module and begin using multiplication to create an equivalent fraction that comprises smaller units, e.g., $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ (**4.NF.1**). Based on the use of multiplication, they reason that division can be used to create a fraction that comprises larger units (or a single unit) equivalent to a given fraction (e.g., $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$). Their work is justified using area models and tape diagrams and, conversely, multiplication is used to test for and/or verify equivalence. Students use the tape diagram to transition to modeling equivalence on the number line.

They see that, by multiplying, any unit fraction length can be partitioned into *n* equal lengths and that doing so multiplies both the total number of fractional units (the denominator) and number of selected units (the numerator) by *n*. They also see that there are times when fractional units can be grouped together, or divided, into larger fractional units. When that occurs, both the total number of fractional units and number of selected units are divided by the same number.

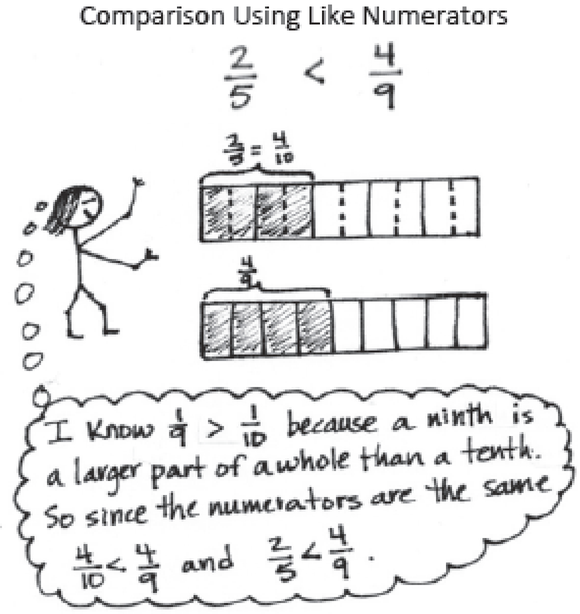


In Grade 3, students compared fractions using fraction strips and number lines with the same denominators. In Topic C, they expand on comparing fractions by reasoning about fractions with unlike denominators. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, $\frac{1}{2}$, or 1) on the number line. Alternatively, students compare using the same numerators. They find that the fraction with the greater denominator is the lesser fraction since the size of the fractional unit is smaller as the whole is decomposed into more equal parts (e.g., $\frac{1}{5} > \frac{1}{10}$; therefore $\frac{3}{5} > \frac{3}{10}$). Throughout the process, their reasoning is supported using tape diagrams and number lines in cases where one numerator or denominator is a factor of the other, such as $\frac{1}{5}$ and $\frac{1}{10}$ or $\frac{2}{3}$ and $\frac{5}{6}$. When the units are unrelated, students use area models and multiplication, the general method pictured below to the left, whereby two fractions are expressed in terms of the same denominators. Students also reason that comparing fractions can only be done when referring to the same whole, and they record their comparisons using the comparison symbols <, >, and = (**4.NF.2**).

Comparison Using Like Denominators

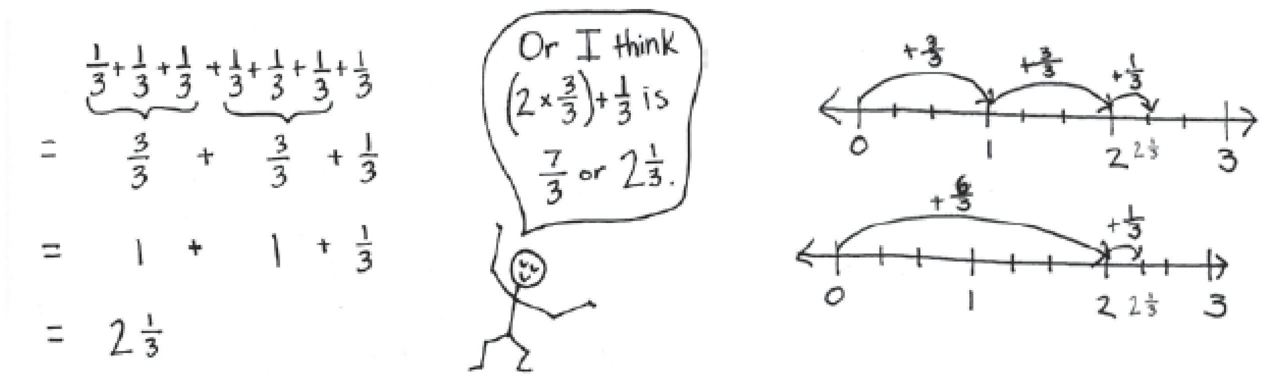


Comparison Using Like Numerators



In Topic D, students apply their understanding of whole number addition (the combining of like units) and subtraction (finding an unknown part) to work with fractions (**4.NF.3a**). They see through visual models that, if the units are the same, computation can be performed immediately, e.g., 2 bananas + 3 bananas = 5 bananas and 2 eighths + 3 eighths = 5 eighths. They see that, when subtracting fractions from one whole, the whole is decomposed into the same units as the part being subtracted, e.g., $1-\frac{3}{5}=\frac{5}{5}-\frac{3}{5}=\frac{2}{5}$. Students practice adding more than two fractions and model fractions in word problems using tape diagrams (**4.NF.3d**). As an extension of the Grade 4 standards, students apply their knowledge of decomposition from earlier topics to add fractions with related units using tape diagrams and area models to support their numerical work. To find the sum of $\frac{1}{2}$ and $\frac{1}{4}$, for example, one simply decomposes 1 half into 2 smaller equal units, fourths, just as in Topics A and B. Now the addition can be completed: $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$. Though not assessed, this work is warranted because, in Module 6, students are asked to add tenths and hundredths when working with decimal fractions and decimal notation.

At the beginning of Topic E, students use decomposition and visual models to add and subtract fractions less than 1 to or from whole numbers, e.g., $4+\frac{3}{4}=4\frac{3}{4}$ and $4-\frac{3}{4}=(3+1)-\frac{3}{4}$. They use addition and multiplication to build fractions greater than 1 and represent them on the number line.



Students then use these visual models and decompositions to reason about the various forms in which a fraction greater than or equal to 1 may be presented, both as fractions and mixed numbers. They practice converting between these forms and begin understanding the usefulness of each form in different situations. Through this understanding, the common misconception that every improper fraction must be converted to a mixed number is avoided. Next, students compare fractions greater than 1, building on their rounding skills and using understanding of benchmarks to reason about which of two fractions is greater (**4.NF.2**). This activity continues to build understanding of the relationship between the numerator and denominator of a fraction. Students progress to finding and using like denominators or numerators to compare and order mixed numbers. They apply their skills of comparing numbers greater than 1 by solving word problems (**4.NF.3d**) requiring the interpretation of data presented in line plots (**4.MD.4**). Students use addition and subtraction strategies to solve the problems, as well as decomposition and modeling to compare numbers in the data sets.

In Topic F, students estimate sums and differences of mixed numbers, rounding before performing the actual operation to determine what a reasonable outcome is. They proceed to use decomposition to add and subtract mixed numbers (**4.NF.3c**). This work builds on their understanding of a mixed number being the sum of a whole number and fraction.

$$3\tfrac{2}{5} + 2\tfrac{4}{5} = 3 + \tfrac{2}{5} + 2 + \tfrac{4}{5} = 3 + 2 + \tfrac{2}{5} + \tfrac{4}{5}$$

I can add the parts in any order without changing the sum.

Using unit form, students add and subtract like units first (e.g., ones and ones, fourths and fourths). Students use decomposition, shown with number bonds, in mixed number addition to make one from fractional units before finding the sum. When subtracting, students learn to decompose the minuend or subtrahend when there are not enough fractional units from which to subtract. Alternatively, students can rename the subtrahend, giving more units to the fractional units, which connects to whole number subtraction when renaming 9 tens 2 ones as 8 tens 12 ones.

$$3\tfrac{1}{5} - \tfrac{3}{5} = 2\tfrac{1}{5} + \tfrac{2}{5} = 2\tfrac{3}{5}$$

$2\tfrac{1}{5}$ 1

Take one out to subtract from one!

$$3\tfrac{1}{5} - \tfrac{3}{5} = 3 - \tfrac{2}{5} = 2\tfrac{3}{5}$$

$\tfrac{1}{5}$ $\tfrac{2}{5}$

Just like subtracting from one!

$$3\tfrac{1}{5} - \tfrac{3}{5} = 2\tfrac{6}{5} - \tfrac{3}{5} = 2\tfrac{3}{5}$$

2 $\tfrac{6}{5}$

Rename to make more fifths!

In Topic G, students build on the concept of representing repeated addition as multiplication, applying this familiar concept to work with fractions (**4.NF.4a**, **4.NF.4b**). They use the associative property and their understanding of decomposition. Just as with whole numbers, the unit remains unchanged.

$$4 \times \frac{3}{5} = 4 \times \left(3 \times \frac{1}{5}\right) = (4 \times 3) \times \frac{1}{5} = \frac{4 \times 3}{5} = \frac{12}{5}$$

This understanding connects to students' work with place value and whole numbers. Students proceed to explore the use of the distributive property to multiply a whole number by a mixed number. They recognize that they are multiplying each part of a mixed number by the whole number and use efficient strategies to do so. The topic closes with solving multiplicative comparison word problems involving fractions (**4.NF.4c**) as well as problems involving the interpretation of data presented on a line plot.

$$\begin{aligned} 5 \times 3\tfrac{3}{4} &= 5 \times \left(3 + \tfrac{3}{4}\right) \\ &= (5 \times 3) + \left(5 \times \tfrac{3}{4}\right) \\ &= 15 + \tfrac{15}{4} \\ &= 15 + 3\tfrac{3}{4} \\ &= 18\tfrac{3}{4} \end{aligned}$$

Topic H comprises an exploration lesson where students find the sum of all like denominators from $\frac{0}{n}$ to $\frac{n}{n}$. Students first work in teams with fourths, sixths, eighths, and tenths. For example, they might find the sum of all sixths from $\frac{0}{6}$ to $\frac{6}{6}$. Students discover that they can make pairs with a sum of 1 to add more efficiently, e.g., $\frac{0}{6} + \frac{6}{6}$, $\frac{1}{6} + \frac{5}{6}$, $\frac{2}{6} + \frac{4}{6}$, and there is one fraction, $\frac{3}{6}$, without a pair. They then extend this to similarly find sums of thirds, fifths, sevenths, and ninths, observing patterns when finding the sum of odd and even denominators (**4.OA.5**).

Terminology

New or Recently Introduced Terms

- Benchmark (standard or reference point by which something is measured)
- Common denominator (when two or more fractions have the same denominator)
- Denominator (e.g., the 5 in 35 names the fractional unit as fifths)
- Fraction greater than 1 (a fraction with a numerator that is greater than the denominator)
- Line plot (display of data on a number line, using an x or another mark to show frequency)
- Mixed number (number made up of a whole number and a fraction)
- Numerator (e.g., the 3 in 35 indicates 3 fractional units are selected)

Familiar Terms and Symbols

- =, <, > (equal to, less than, greater than)
- Compose (change a smaller unit for an equivalent of a larger unit, e.g., 2 fourths = 1 half, 10 ones = 1 ten; combining 2 or more numbers, e.g., 1 fourth + 1 fourth = 2 fourths, 2 + 2 + 1 = 5)
- Decompose (change a larger unit for an equivalent of a smaller unit, e.g., 1 half = 2 fourths, 1 ten = 10 ones; partition a number into 2 or more parts, e.g., 2 fourths = 1 fourth + 1 fourth, 5 = 2 + 2 + 1)
- Equivalent fractions (fractions that name the same size or amount)
- Fraction (e.g. $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}$)
- Fractional unit (e.g., half, third, fourth)
- Multiple (product of a given number and any other whole number)
- Non-unit fraction (fractions with numerators other than 1)
- Unit fraction (fractions with numerator 1)
- Unit interval (e.g., the interval from 0 to 1, measured by length)
- Whole (e.g., 2 halves, 3 thirds, 4 fourths)

Suggested Tools and Representations

- Area model
- Fraction strips (made from paper, folded, and used to model equivalent fractions)
- Line plot
- Number line
- Rulers
- Tape diagram

Grade 4 Module 5 Topic A

Decomposition and Fraction Equivalence

Focus Standards:

4.NF.3b Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

4.NF.4a Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

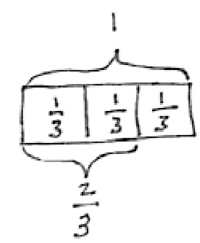
a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

Instructional Days Recommended: 6

Topic A builds on Grade 3 work with unit fractions. Students explore fraction equivalence through the decomposition of non-unit fractions into unit fractions, as well as the decomposition of unit fractions into smaller unit fractions. They represent these decompositions, and prove equivalence, using visual models.

In Lesson 1, students use paper strips to represent the decomposition of a whole into parts. In Lessons 1 and 2, students decompose fractions as unit fractions, drawing tape diagrams to represent them as sums of fractions with the same denominator in different ways, e.g., $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1}{5} + \frac{2}{5}$

$$1 = \underbrace{\frac{1}{3} + \frac{1}{3}} + \frac{1}{3}$$
$$= \frac{2}{3} + \frac{1}{3}$$
$$= \frac{3}{3}$$



In Lesson 3, students see that representing a fraction as the repeated addition of a unit fraction is the same as multiplying that unit fraction by a whole number. This is already a familiar fact in other contexts.
An example is as follows:

3 bananas = 1 banana + 1 banana + 1 banana = 3 × 1 banana,

3 twos = 2 + 2 + 2 = 3 × 2

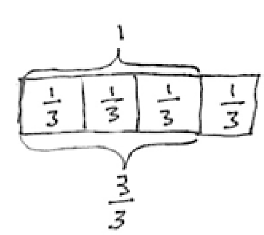
3 fourths = 1 fourth + 1 fourth + 1 fourth = 3 × 1 fourth,

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3\,x\,\frac{1}{4}$$

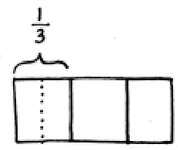
By introducing multiplication as a record of the decomposition of a fraction early in the module, students are accustomed to the notation by the time they work with more complex problems in Topic G.

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$
$$= 2 \times \frac{1}{3}$$

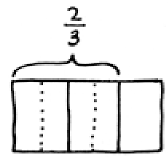
$$\frac{4}{3} = \frac{3}{3} + \frac{1}{3}$$
$$= 1\,\frac{1}{3}$$
$$= 4 \times \frac{1}{3}$$



Students continue with decomposition in Lesson 4, where they use tape diagrams to represent fractions, e.g., $\frac{1}{2}, \frac{1}{3}$, and $\frac{2}{3}$, as the sum of smaller unit fractions. Students record the results as a number sentence, e.g., $\frac{1}{2} = \frac{1}{4} + \frac{1}{4} = (\frac{1}{8} + \frac{1}{8}) + (\frac{1}{8} + \frac{1}{8}) = \frac{4}{8}$

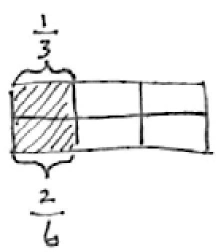


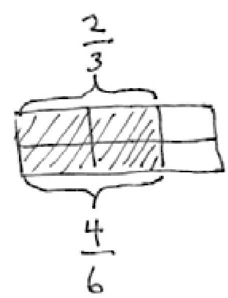
$$\frac{1}{3} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$



$$\frac{2}{3} = \left(\frac{1}{6} + \frac{1}{6}\right) + \left(\frac{1}{6} + \frac{1}{6}\right) = \frac{4}{6}$$

In Lesson 5, this idea is further investigated as students represent the decomposition of unit fractions in area models. In Lesson 6, students use the area model for a second day, this time to represent fractions with different numerators. They explain why two different fractions represent the same portion of a whole.





**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 1 - 2

Objective: Decompose fractions as a sum of unit fractions using tape diagrams.

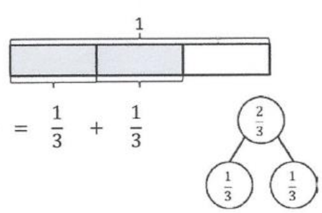
Homework Key (1)

1. a. Answer provided
   b. Whole: $\frac{2}{4}$, parts: $\frac{1}{4}, \frac{1}{4}$; $\frac{2}{4} = \frac{1}{4} + \frac{1}{4}$
   c. Whole: $\frac{3}{5}$, parts: $\frac{1}{5}, \frac{2}{5}$; $\frac{3}{5} = \frac{1}{5} + \frac{2}{5}$
   d. Whole: $\frac{5}{6}$, parts: $\frac{3}{6}, \frac{2}{6}$; $\frac{5}{6} = \frac{3}{6} + \frac{2}{6}$
   e. Whole: $\frac{3}{8}$, parts: $\frac{2}{8}, \frac{1}{8}$; $\frac{3}{8} = \frac{2}{8} + \frac{1}{8}$
   f. Whole: $1\frac{1}{5}$, parts: $\frac{5}{5}, \frac{1}{5}$; $1\frac{1}{5} = \frac{5}{5} + \frac{1}{5}$
   g. Whole: $1\frac{2}{4}$, parts: $\frac{3}{4}, \frac{2}{4}, \frac{1}{4}$; $1\frac{2}{4} = \frac{3}{4} + \frac{2}{4} + \frac{1}{4}$
   h. Whole: $1\frac{4}{8}$, parts: $\frac{3}{8}, \frac{2}{8}, \frac{1}{8}, \frac{3}{8}, \frac{3}{8}$; $1\frac{4}{8} = \frac{3}{8} + \frac{2}{8} + \frac{1}{8} + \frac{3}{8} + \frac{3}{8}$
2. a. Tape diagram models number sentence.
   b. Tape diagram models number sentence.
   c. Tape diagram models number sentence.
   d. Tape diagram models number sentence.
   e. Tape diagram models number sentence.
   f. Tape diagram models number sentence.

Homework Samples

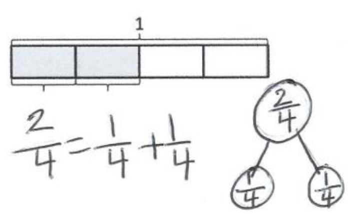
1. Draw a number bond and write the number sentence to match each tape diagram. The first one is done for you.

a.



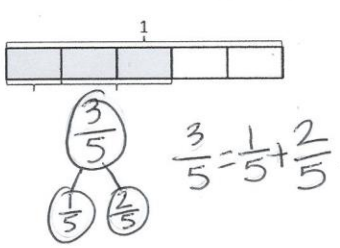
$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$

b.



$\frac{2}{4} = \frac{1}{4} + \frac{1}{4}$

c.

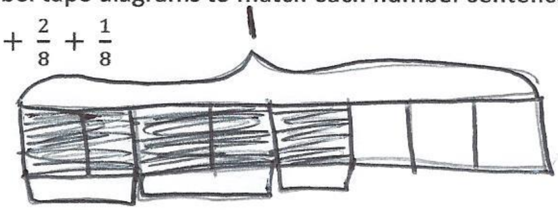


$\frac{3}{5} = \frac{1}{5} + \frac{2}{5}$

Lesson 1 (continued)

2. Draw and label tape diagrams to match each number sentence.

   a. $\frac{5}{8} = \frac{2}{8} + \frac{2}{8} + \frac{1}{8}$



Lesson 2

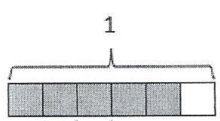
Homework Key

1. a. Answer provided.
   b. Tape diagram models number sentence; decompositions will vary.
   c. Tape diagram models number sentence; decompositions will vary.
2. a. Tape diagram models number sentence; decompositions will vary.
   b. Tape diagram models number sentence; decompositions will vary.
   c. Tape diagram models number sentence; decompositions will vary.
   d. Tape diagram models number sentence; decompositions will vary.

Homework Sample

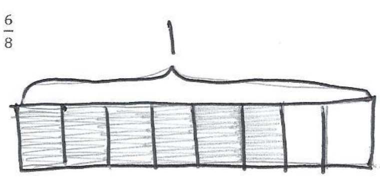
1. Step 1: Draw and shade a tape diagram of the given fraction.
   Step 2: Record the decomposition as a sum of unit fractions.
   Step 3: Record the decomposition of the fraction two more ways.
   (The first one has been done for you.)

   a. $\frac{5}{6}$



$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ $\quad$ $\frac{5}{6} = \frac{2}{6} + \frac{2}{6} + \frac{1}{6}$ $\quad$ $\frac{5}{6} = \frac{1}{6} + \frac{4}{6}$

   b. $\frac{6}{8}$



$\frac{6}{8} = \frac{2}{8} + \frac{2}{8} + \frac{2}{8}$ $\quad$ $\frac{6}{8} = \frac{3}{8} + \frac{3}{8}$ $\quad$ $\frac{6}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

Lesson 3

Objective: Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.

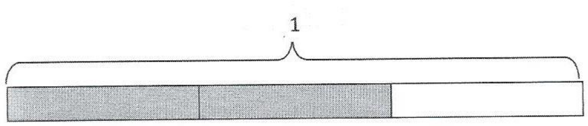
Homework Key

1. a. Answer provided.
   b. $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$; $\frac{3}{4} = 3 \times \frac{1}{4}$
   c. $\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$; $\frac{4}{5} = 4 \times \frac{1}{5}$
   d. $\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$; $\frac{5}{6} = 5 \times \frac{1}{6}$
2. a. $\frac{4}{3} = \left(3 \times \frac{1}{3}\right) + \left(1 \times \frac{1}{3}\right)$
   b. $\frac{8}{6} = \left(6 \times \frac{1}{6}\right) + \left(2 \times \frac{1}{6}\right)$
3. a. Tape diagram models number sentence; $\frac{3}{5} = 3 \times \frac{1}{5}$
   b. Tape diagram models number sentence; $\frac{3}{8} = 3 \times \frac{1}{8}$
   c. Tape diagram models number sentence; $\frac{5}{9} = 5 \times \frac{1}{9}$
   d. Tape diagram models number sentence $\frac{8}{5} = 8 \times \frac{1}{5}$
   e. Tape diagram models number sentence; $\frac{12}{4} = 12 \times \frac{1}{4}$

Homework Samples

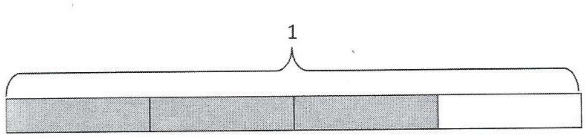
1. Decompose each fraction modeled by a tape diagram as a sum of unit fractions. Write the equivalent multiplication sentence. The first one has been done for you.

a.



$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$ $\frac{2}{3} = 2 \times \frac{1}{3}$

b.

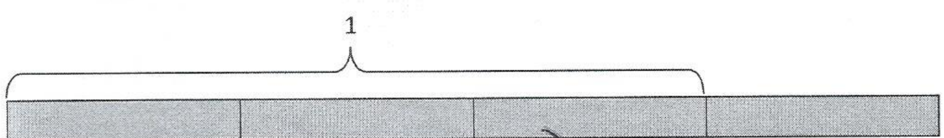


$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ $\frac{3}{4} = 3 \times \frac{1}{4}$

Lesson 3 (continued)

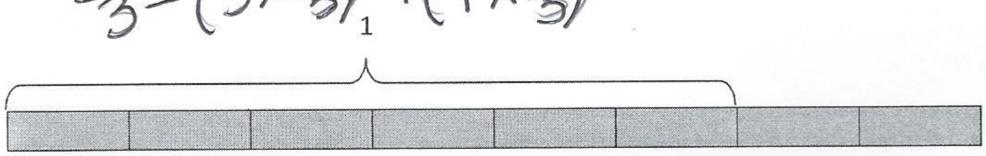
2. Write the following fractions greater than 1 as the sum of two products.

a.



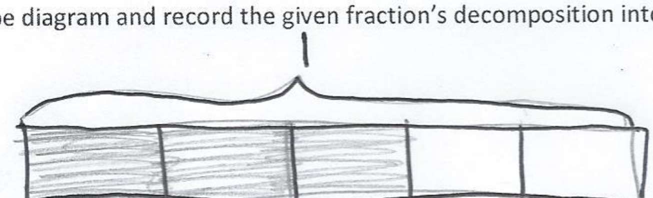
$\frac{4}{3} = (3 \times \frac{1}{3}) + (1 \times \frac{1}{3})$

b.



3. Draw a tape diagram and record the given fraction's decomposition into unit fractions as a multiplication sentence.

a. $\frac{3}{5}$



$\frac{3}{5} = 3 \times \frac{1}{5}$

b. $\frac{3}{8}$

Lesson 4

Objective: Decompose fractions into sums of smaller unit fractions using tape diagrams.

Homework Key

1. a. Answer provided.

   b. $\frac{1}{4} = \frac{1}{8} + \frac{1}{8}$; $\frac{1}{4} = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$

2. a. Answers will vary.

   b. Answers will vary.

   c. Answers will vary.

3. a. Answer provided.

   b. Tape diagram models number sentence.

   c. Tape diagram models number sentence.

   d. Tape diagram models number sentence.

4. Tape diagram models number sentence.

5. Tape diagram models number sentence.

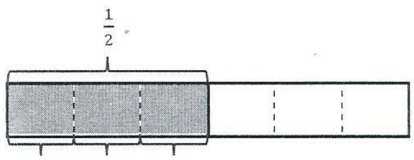
6. Tape diagram models number sentence.

Lesson 4 (continued)

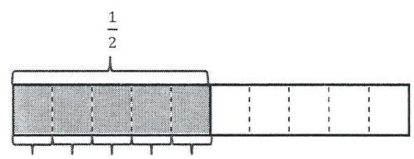
Homework Sample

1. The total length of each tape diagram represents 1. Decompose the shaded unit fractions as the sum of smaller unit fractions in at least two different ways. The first one has been done for you.

a.

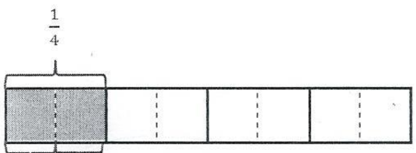


$\frac{1}{2} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

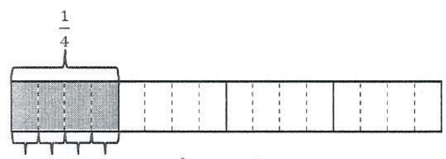


$\frac{1}{2} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$

b.



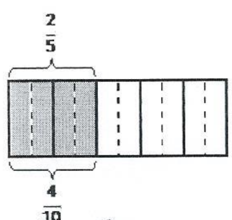
$\frac{1}{4} = \frac{1}{8} + \frac{1}{8}$



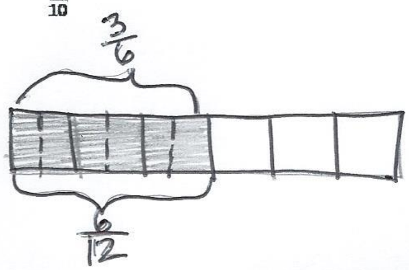
$\frac{1}{4} = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}$

3. Draw tape diagrams to prove the following statements. The first one has been done for you.

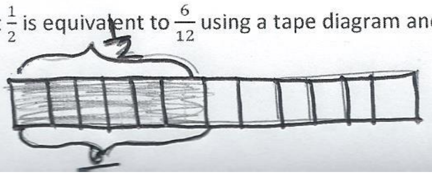
a. $\frac{2}{5} = \frac{4}{10}$



b. $\frac{3}{6} = \frac{6}{12}$



4. Show that $\frac{1}{2}$ is equivalent to $\frac{6}{12}$ using a tape diagram and a number sentence.



$\frac{1}{2} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$

Lesson 5

Objective: Decompose unit fractions using area models to show equivalence.

Homework Key

1. a. $6, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}$

   b. 2 rows drawn; $\frac{1}{4} = \frac{2}{8}, \frac{1}{4} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8}, \frac{1}{4} = 2 \times \frac{1}{8} = \frac{2}{8}$

   c. 4 rows drawn; $\frac{1}{4} = \frac{4}{16}, \frac{1}{4} = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16}, \frac{1}{4} = 4 \times \frac{1}{16} = \frac{4}{16}$

2. a. Area model shows $\frac{1}{3} = \frac{2}{6}; \frac{1}{3} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}, \frac{1}{3} = 2 \times \frac{1}{6} = \frac{2}{6}$

   b. Area model shows $\frac{1}{3} = \frac{3}{9}; \frac{1}{3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9}, \frac{1}{3} = 3 \times \frac{1}{9} = \frac{3}{9}$

   c. Area model shows $\frac{1}{3} = \frac{4}{12}; \frac{1}{3} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12}, \frac{1}{3} = 4 \times \frac{1}{12} = \frac{4}{12}$

   d. Area model shows $\frac{1}{3} = \frac{5}{15}; \frac{1}{3} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{5}{15}, \frac{1}{3} = 5 \times \frac{1}{15} = \frac{5}{15}$

   e. Area model shows $\frac{1}{5} = \frac{2}{10}; \frac{1}{5} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}, \frac{1}{5} = 2 \times \frac{1}{10} = \frac{2}{10}$

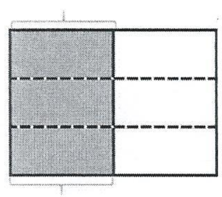
   f. Area model shows $\frac{1}{5} = \frac{3}{15}; \frac{1}{5} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15}, \frac{1}{5} = 3 \times \frac{1}{15} = \frac{3}{15}$

3. Explanations will vary.

Homework Samples

1. Draw horizontal lines to decompose each rectangle into the number of rows as indicated. Use the model to give the shaded area as both a sum of unit fractions and as a multiplication sentence.

   a. 3 rows

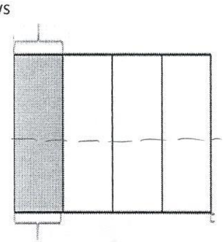


$\frac{1}{2} = \frac{3}{6}$

$\frac{1}{2} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$

$\frac{1}{2} = 3 \times \frac{1}{6} = \frac{3}{6}$

   b. 2 rows



$\frac{1}{4} = \frac{2}{8}$

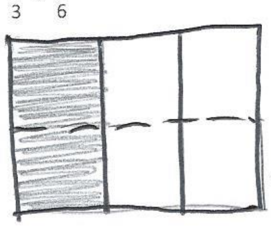
$\frac{1}{4} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8}$

$\frac{1}{4} = 2 \times \frac{1}{8} = \frac{2}{8}$

Lesson 5 (continued)

2. Draw area models to show the decompositions represented by the number sentences below. Represent the decomposition as a sum of unit fractions and as a multiplication sentence.

a. $\frac{1}{3} = \frac{2}{6}$



$\frac{1}{3} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

$\frac{1}{3} = 2 \times \frac{1}{6} = \frac{2}{6}$

b. $\frac{1}{3} = \frac{3}{9}$

Lesson 6

Objective: Decompose fractions using area models to show equivalence.

Homework Key

1. a. 4, 10, 1, 1, 10, $\frac{1}{10}$, $\frac{1}{10}$, 10, $\frac{1}{10}$, 10

   b. Decomposed horizontally to show eighths; $\frac{1}{4} + \frac{1}{4} = \left(\frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{8} + \frac{1}{8}\right) = \frac{4}{8}$, $\left(\frac{1}{8} + \frac{1}{8}\right) + \left(\frac{1}{8} + \frac{1}{8}\right) = \left(2 \times \frac{1}{8}\right) + \left(2 \times \frac{1}{8}\right) = \frac{4}{8}$, $\frac{2}{4} = 4 \times \frac{1}{8} = \frac{4}{8}$

   c. Decomposed horizontally to show fifteenths; $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{15}\right) + \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{15}\right) + \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{15}\right) + \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{15}\right) = \frac{12}{15}$; $\left(\frac{1}{15} + \frac{1}{15} + \frac{1}{15}\right) + \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{15}\right) + \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{15}\right) + \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{15}\right) = \left(3 \times \frac{1}{15}\right) + \left(3 \times \frac{1}{15}\right) + \left(3 \times \frac{1}{15}\right) + \left(3 \times \frac{1}{15}\right) = \frac{12}{15}$, $\frac{4}{5} = 12 \times \frac{1}{15} = \frac{12}{15}$

2. a. Area model shows $\frac{2}{3} = \frac{4}{6}$; $\frac{1}{3} + \frac{1}{3} = \left(\frac{1}{6} + \frac{1}{6}\right) + \left(\frac{1}{6} + \frac{1}{6}\right) = \frac{4}{6}$, $\left(\frac{1}{6} + \frac{1}{6}\right) + \left(\frac{1}{6} + \frac{1}{6}\right) = \left(2 \times \frac{1}{6}\right) +$

   b. $\left(2 \times \frac{1}{6}\right) = \frac{4}{6}$, $\frac{2}{3} = 4 \times \frac{1}{6} = \frac{4}{6}$

   c. Area model shows $\frac{4}{5} = \frac{8}{10}$; $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \left(\frac{1}{10} + \frac{1}{10}\right) + \left(\frac{1}{10} + \frac{1}{10}\right) + \left(\frac{1}{10} + \frac{1}{10}\right) + \left(\frac{1}{10} + \frac{1}{10}\right) = \frac{8}{10}$,

   d. $\left(\frac{1}{10} + \frac{1}{10}\right) + \left(\frac{1}{10} + \frac{1}{10}\right) + \left(\frac{1}{10} + \frac{1}{10}\right) + \left(\frac{1}{10} + \frac{1}{10}\right) = \left(2 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{10}\right) +$

   e. $\left(2 \times \frac{1}{10}\right) = \frac{8}{10}$, $\frac{4}{5} = 6 \times \frac{1}{10} = \frac{6}{10}$

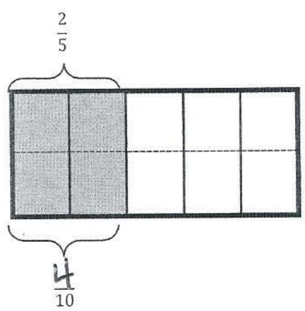
3. Answers will vary.

Lesson 6 (continued)

Homework Samples

1. Each rectangle represents 1. Draw horizontal lines to decompose each rectangle into the fractional units as indicated. Use the model to give the shaded area as a sum and as a product of unit fractions. Use parentheses to show the relationship between the number sentences. The first one has been partially done for you.

   a. Tenths



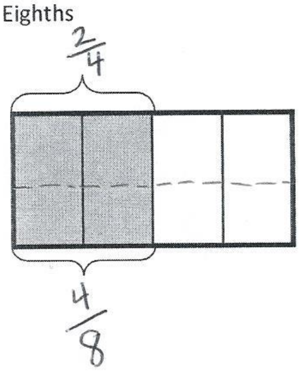
$$\frac{2}{5} = \frac{4}{10}$$

$$\frac{1}{5}+\frac{1}{5} = \left(\frac{1}{10}+\frac{1}{10}\right)+\left(\frac{1}{10}+\frac{1}{10}\right) = \frac{4}{10}$$

$$\left(\frac{1}{10}+\frac{1}{10}\right)+\left(\frac{1}{10}+\frac{1}{10}\right) = \left(2\times\frac{1}{10}\right)+\left(2\times\frac{1}{10}\right) = \frac{4}{10}$$

$$\frac{2}{5} = 4\times\frac{1}{10} = \frac{4}{10}$$

   b. Eighths



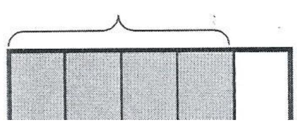
$$\frac{2}{4} = \frac{4}{8}$$

$$\frac{1}{4}+\frac{1}{4} = \left(\frac{1}{8}+\frac{1}{8}\right)+\left(\frac{1}{8}+\frac{1}{8}\right) = \frac{4}{8}$$

$$\left(\frac{1}{8}+\frac{1}{8}\right)+\left(\frac{1}{8}+\frac{1}{8}\right) = \left(2\times\frac{1}{8}\right)+\left(2\times\frac{1}{8}\right) = \frac{4}{8}$$

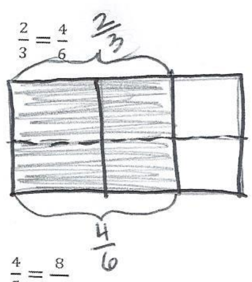
$$\frac{2}{4} = 4\times\frac{1}{8} = \frac{4}{8}$$

   c. Fifteenths



2. Draw area models to show the decompositions represented by the number sentences below. Express each as a sum and product of unit fractions. Use parentheses to show the relationship between the number sentences.

   a. $\frac{2}{3} = \frac{4}{6}$



$$\frac{1}{3}+\frac{1}{3} = \left(\frac{1}{6}+\frac{1}{6}\right)+\left(\frac{1}{6}+\frac{1}{6}\right) = \frac{4}{6}$$

$$\left(\frac{1}{6}+\frac{1}{6}\right)+\left(\frac{1}{6}+\frac{1}{6}\right) = \left(2\times\frac{1}{6}\right)+\left(2\times\frac{1}{6}\right) = \frac{4}{6}$$

$$\frac{2}{3} = \left(4\times\frac{1}{6}\right) = \frac{4}{6}$$

   b. $\frac{4}{5} = \frac{8}{10}$

Grade 4 Module 5 Topic B

Fraction Equivalence Using Multiplication and Division

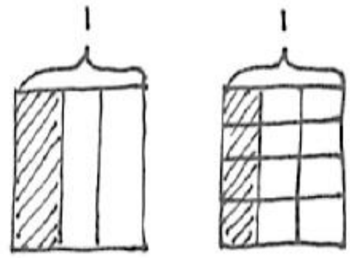
Focus Standard:

4.NF.1 Explain why a fraction *a/b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Instructional Days Recommended: 5

In Topic B, students begin generalizing their work with fraction equivalence. In Lessons 7 and 8, students analyze their earlier work with tape diagrams and the area model in Lessons 3 through 5 to begin using multiplication to create an equivalent fraction that comprises smaller units, e.g., $\frac{2}{3} = \frac{2 \; x \; 3}{3 \; x \; 4} = \frac{8}{12}$. Conversely, students reason, in Lessons 9 and 10, that division can be used to create a fraction that comprises larger units (or a single unit) equivalent to a given fraction, e.g., $\frac{2}{3}$ $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$. The numerical work of Lessons 7 through 10 is introduced and supported using area models and tape diagrams.

In Lesson 11, students use tape diagrams to transition their knowledge of fraction equivalence to the number line. They see that any unit fraction length can be partitioned into *n* equal lengths. For example, each third in the interval from 0 to 1 may be partitioned into 4 equal parts. Doing so multiplies both the total number of fractional units (the denominator) and the number of selected units (the numerator) by 4. Conversely, students see that, in some cases, fractional units may be grouped together to form some number of larger fractional units. For example, when the interval from 0 to 1 is partitioned into twelfths, one may group 4 twelfths at a time to make thirds. By doing so, both the total number of fractional units and number of selected units are divided by 4.



$$\frac{1}{3} = \frac{4 \times 1}{4 \times 3}$$
$$= \frac{4}{12}$$

1 third = 4 twelfths

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 7 - 8

Objective: Use the area model and multiplication to show the equivalence of two fractions.

Homework Key (7)

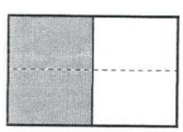
1. a. Answer provided
   b. $\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$
   c. $\frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12}$
   d. $\frac{1}{2} = \frac{1 \times 7}{2 \times 7} = \frac{7}{14}$
2. a. Answers will vary.
   b. Answers will vary.
   c. Answers will vary.
   d. Answers will vary.
3. a. Area model shows $\frac{1}{4}$ and is decomposed horizontally into eighths; $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$
   b. Area model shows $\frac{1}{4}$ and is decomposed horizontally into twelfths; $\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$
   c. Area model shows $\frac{1}{4}$ and is decomposed horizontally into sixteenths; $\frac{1}{4} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$

Homework Samples (7)

Each rectangle represents 1.

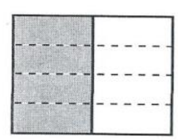
1. The shaded unit fractions have been decomposed into smaller units. Express the equivalent fractions in a number sentence using multiplication. The first one has been done for you.

a.



$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$

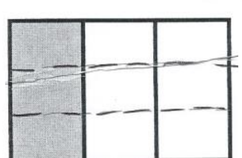
b.



$\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

2. Decompose the shaded fractions into smaller units using the area models. Express the equivalent fractions in a number sentence using multiplication.

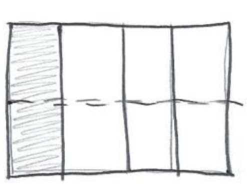
a.



$\frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9}$

3. Draw three different area models to represent 1 fourth by shading.
Decompose the shaded fraction into (a) eighths, (b) twelfths, and (c) sixteenths.
Use multiplication to show how each fraction is equivalent to 1 fourth.

a.



$\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$

Lesson 8

Homework Key

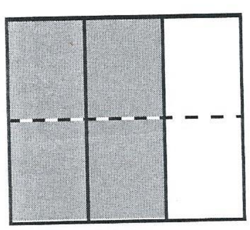
1. a. Answer provided
   b. $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$
   c. $\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$
   d. $\frac{7}{8} = \frac{7 \times 2}{8 \times 2} = \frac{14}{16}$
2. a. $\frac{3}{6} = \frac{3 \times 2}{6 \times 2} = \frac{6}{12}$
   b. $\frac{2}{4} = \frac{2 \times 3}{4 \times 3} = \frac{6}{12}$
3. a. Area model proves $\frac{1}{3} = \frac{2}{6}$
   b. Area model proves $\frac{2}{5} = \frac{4}{10}$
   c. Area model proves $\frac{5}{7} = \frac{10}{14}$
   d. Area model proves $\frac{3}{6} = \frac{9}{18}$
4. a. Answers will vary.
   b. Answers will vary.
   c. Answers will vary.
   d. Answers will vary.
5. a. False; answers will vary.
   b. True
   c. False; answers will vary.
   d. True

Homework Samples

Each rectangle represents 1.

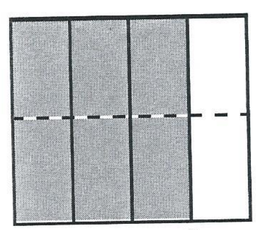
1. The shaded fractions have been decomposed into smaller units. Express the equivalent fractions in a number sentence using multiplication. The first one has been done for you.

a.



$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

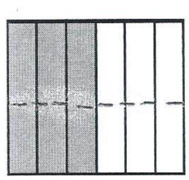
b.



$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

2. Decompose both shaded fractions into twelfths. Express the equivalent fractions in a number sentence using multiplication.

a.

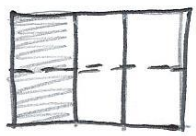


$\frac{3}{6} = \frac{3 \times 2}{6 \times 2} = \frac{6}{12}$

Lesson 8 (continued)

3. Draw area models to prove that the following number sentences are true.

   a. $\frac{1}{3} = \frac{2}{6}$



$\frac{1}{3} = \frac{2}{6}$

4. Use multiplication to create an equivalent fraction for each fraction below.

   a. $\frac{2}{3}$

$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

5. Determine which of the following are true number sentences. Correct those that are false by changing the right-hand side of the number sentence.

   a. $\frac{2}{3} = \frac{4}{6}$ False

Multiply by 2
$2 \times 2 = 4, 3 \times 2 = 6$

Lesson 9 - 10

Objective: Use the area model and division to show the equivalence of two fractions.

Homework Key (9)

1. a. Answer provided

   b. Model shows $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$ or $\frac{4}{8} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4}$

   c. Model shows $\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$ or $\frac{6}{12} = \frac{6 \div 3}{12 \div 3} = \frac{2}{4}$ or $\frac{6}{12} = \frac{6 \div 2}{12 \div 2} = \frac{3}{6}$

   d. Model shows $\frac{7}{14} = \frac{7 \div 7}{14 \div 7} = \frac{1}{2}$

2. a. Model shows $\frac{2}{12} = \frac{2 \div 2}{12 \div 2} = \frac{1}{6}$

   b. Model shows $\frac{2}{10} = \frac{2 \div 2}{10 \div 2} = \frac{1}{5}$

   c. Model shows $\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$

   d. Model shows $\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$

   e. The size of the fractional units increased.

   f. The number of total units decreased.

3. a. Area models prove $\frac{4}{8} = \frac{1}{2}$ and $\frac{6}{12} = \frac{1}{2}$

   b. $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}, \frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$

4. a. Area models prove $\frac{4}{8} = \frac{1}{2}$ and $\frac{8}{16} = \frac{1}{2}$

   b. $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}, \frac{8}{16} = \frac{8 \div 8}{16 \div 8} = \frac{1}{2}$

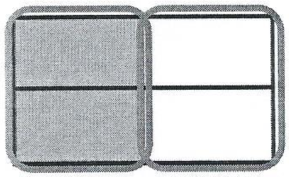
Lesson 9 (continued)

Homework Samples

Each rectangle represents 1.

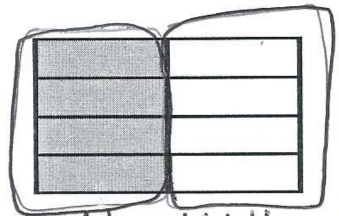
1. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division. The first one has been done for you.

a.



$$\frac{2}{4}=\frac{2\div 2}{4\div 2}=\frac{1}{2}$$

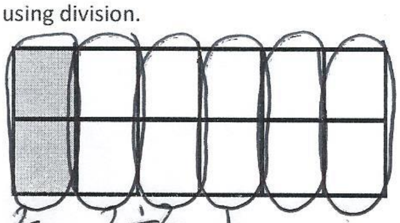
b.



$$\frac{4}{8}=\frac{4\div 4}{8\div 4}=\frac{1}{2}$$

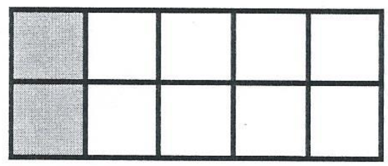
2. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

a.

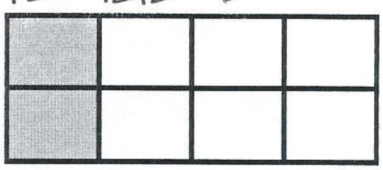


$$\frac{2}{12}=\frac{2\div 2}{12\div 2}=\frac{1}{6}$$

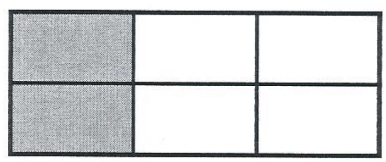
b.



c.

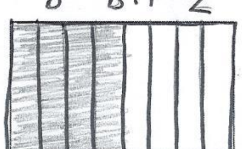


d.

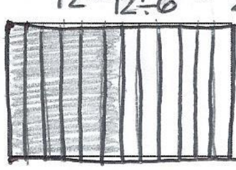


3. a. In the first area model, show 4 eighths. In the second area model, show 6 twelfths. Show how both fractions can be composed, or renamed, as the same unit fraction.

$$\frac{4}{8}=\frac{4\div 4}{8\div 4}=\frac{1}{2}$$



$$\frac{6}{12}=\frac{6\div 6}{12\div 6}=\frac{1}{2}$$



Lesson 10

Homework Key

1. a. Answer provided

   b. Area model shows composed fractions; $\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

   c. Area model shows composed fractions; $\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

   d. Area model shows composed fractions; $\frac{9}{15} = \frac{9 \div 3}{15 \div 3} = \frac{3}{5}$

2. a. Area model shows composed fractions; $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

   b. Area model shows composed fractions; $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$ or $\frac{12}{16} = \frac{12 \div 2}{16 \div 2} = \frac{6}{8}$

3. a. Area model shows $\frac{6}{15}$ composed as $\frac{2}{5}$

   b. Area model shows $\frac{6}{18}$ composed as $\frac{2}{6}$

4. a. $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$

   b. $\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

   c. $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

   d. $\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$

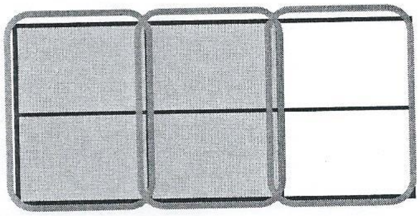
Lesson 10 (continued)

Homework Samples

Each rectangle represents 1.

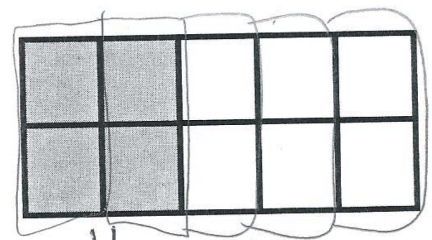
1. Compose the shaded fraction into larger fractional units. Express the equivalent fractions in a number sentence using division. The first one has been done for you.

a.



$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

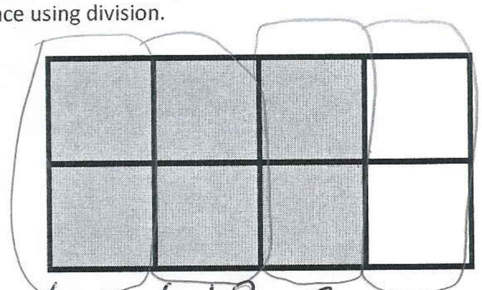
b.



$\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

2. Compose the shaded fractions into larger fractional units. Express the equivalent fractions in a number sentence using division.

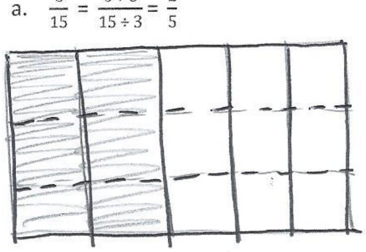
a.



$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

3. Draw an area model to represent each number sentence below.

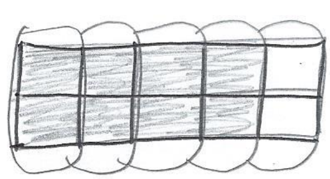
a. $\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$



b. $\frac{6}{18} = \frac{6 \div 3}{18 \div 3} = \frac{2}{6}$

4. Use division to rename each fraction given below. Draw a model if that helps you. See if you can use the largest common factor.

a. $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$



Lesson 11

Objective: Explain fraction equivalence using a tape diagram and the number line, and relate that to the use of multiplication and division.

Homework Key

1. a. $\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}; \frac{1}{3}$ circled

   b. $\frac{0}{6}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}; \frac{2}{6}$ circled

   c. $\frac{0}{12}, \frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{4}{12}, \frac{5}{12}, \frac{6}{12}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}, \frac{11}{12}, \frac{12}{12}; \frac{4}{12}$ circled

2. a. $\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

   b. $\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$

3. a. Number line drawn for $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}; \frac{2}{4}$ circled

   b. Number line drawn for $\frac{0}{8}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}; \frac{4}{8}$ circled

   c. Number line drawn for $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}; \frac{5}{10}$ circled

4. $\frac{4}{8} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4}$

5. a. Number line drawn appropriately

   b. $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

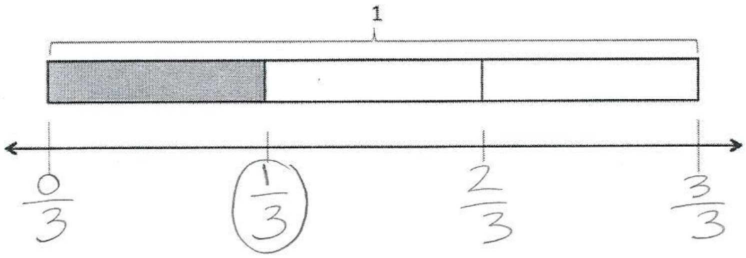
   c. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

Lesson 11 (continued)

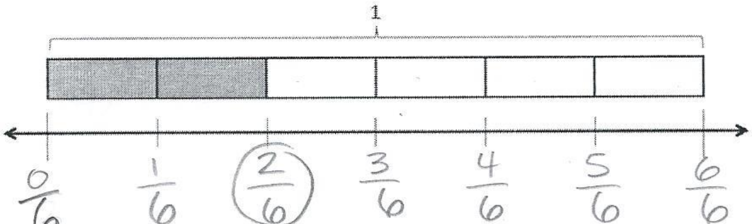
Homework Samples

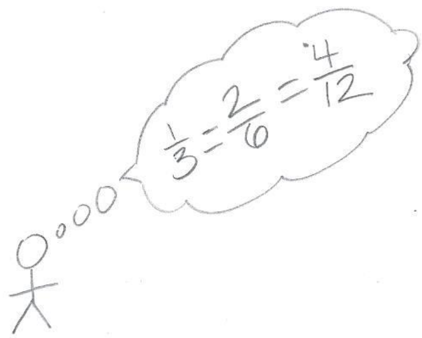
1. Label each number line with the fractions shown on the tape diagram. Circle the fraction that labels the point on the number line that also names the selected part of the tape diagram.

a.

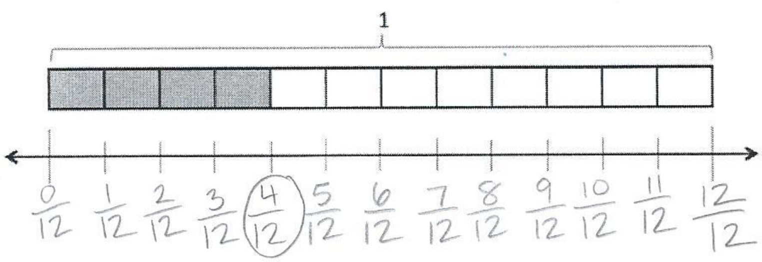


b.





c.



2. Write number sentences using multiplication to show:
   a. The fraction represented in 1(a) is equivalent to the fraction represented in 1(b).

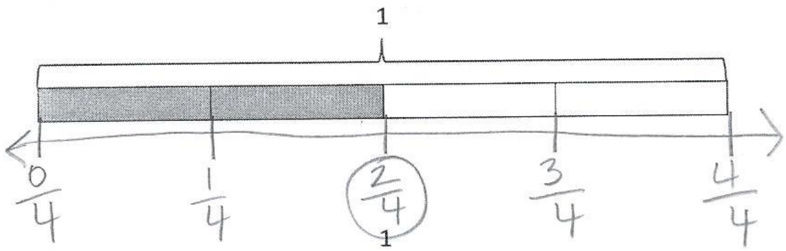
$$\frac{1}{3}=\frac{1\times2}{3\times2}=\frac{2}{6}$$

Lesson 11 (continued)

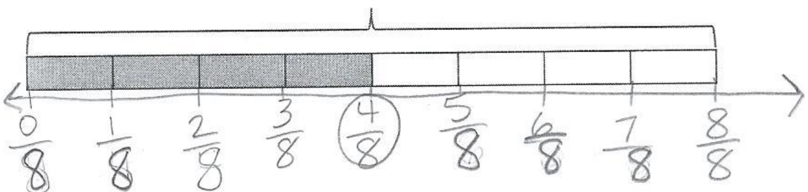
Homework Samples

3. Use each shaded tape diagram below as a ruler to draw a number line. Mark each number line with the fractional units shown on the tape diagram, and circle the fraction that labels the point on the number line that also names the selected part of the tape diagram.

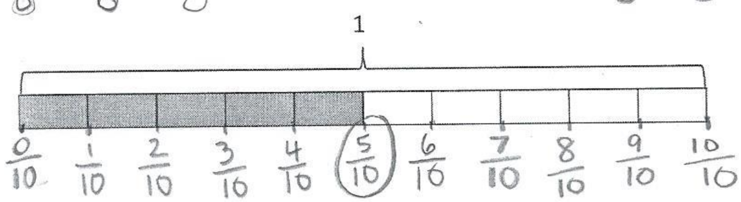
a.



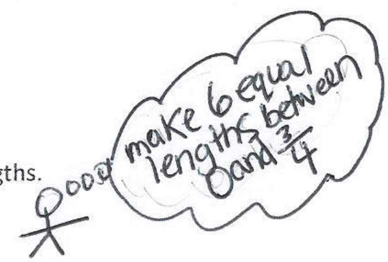
b.



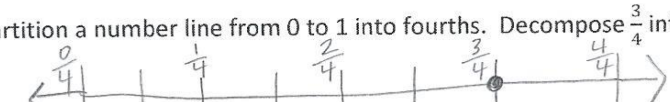
c.



4. Write a number sentence using division to show the fraction represented in 3(a) is equivalent to the fraction represented in 3(b).



5. a. Partition a number line from 0 to 1 into fourths. Decompose $\frac{3}{4}$ into 6 equal lengths.



b. Write a number sentence using multiplication to show what fraction represented on the number line is equivalent to $\frac{3}{4}$.

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

c. Write a number sentence using division to show what fraction represented on the number line is equivalent to $\frac{3}{4}$.

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

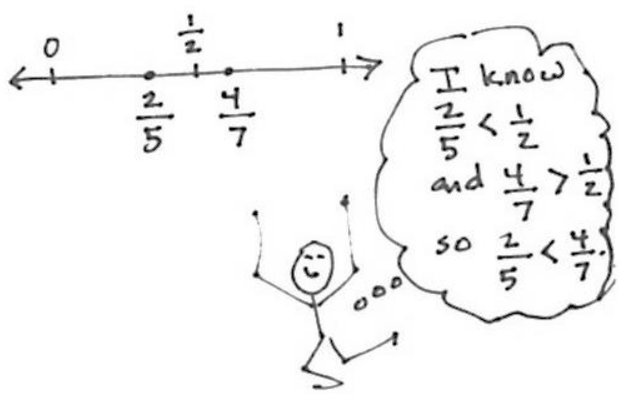
Fraction Comparison

Focus Standard:

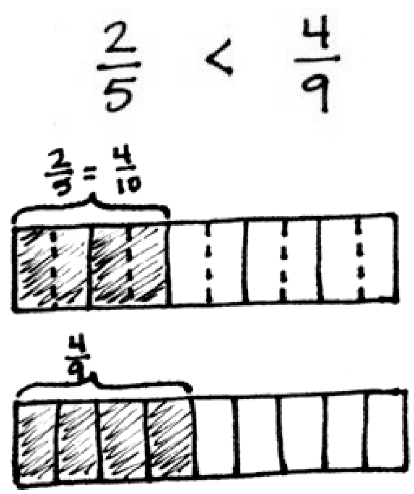
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Instructional Days Recommended: 4

In Topic C, students use benchmarks and common units to compare fractions with different numerators and different denominators. The use of benchmarks is the focus of Lessons 12 and 13 and is modeled using a number line. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, $\frac{1}{2}$, or 1) and then use that information to compare the given fractions. For example, when comparing $\frac{4}{7}$ and $\frac{2}{5}$, students reason that 4 sevenths is more than 1 half, while 2 fifths is less than 1 half. They then conclude that 4 sevenths is greater than 2 fifths.



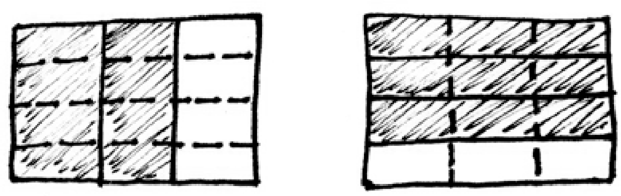
In Lesson 14, students reason that they can also use like numerators based on what they know about the size of the fractional units. They begin at a simple level by reasoning, for example, that 3 fifths is less than 3 fourths because fifths are smaller than fourths. They then see, too, that it is easy to make like numerators at times to compare, e.g., $\frac{2}{5} < \frac{4}{9}$ because $\frac{2}{5} = \frac{4}{10}$, and $\frac{4}{10} < \frac{4}{9}$ because $\frac{1}{10} < \frac{1}{9}$. Using their experience with fractions in Grade 3, they know the larger the denominator of a unit fraction, the smaller the size of the fractional unit.



Like numerators are modeled using tape diagrams directly above each other, where one fractional unit is partitioned into smaller unit fractions. The lesson then moves to comparing fractions with related denominators, such as $\frac{2}{3}$ and $\frac{5}{6}$, wherein one denominator is a factor of the other, using both tape diagrams and the number line. In Lesson 15, students compare fractions by using an area model to express two fractions, wherein one denominator is not a factor of the other, in terms of the same unit using multiplication, e.g., $\frac{2}{3} < \frac{3}{4}$ because $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ and $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ and $\frac{8}{12} < \frac{9}{12}$. The area for $\frac{2}{3}$ is partitioned vertically, and the area for $\frac{3}{4}$ is partitioned horizontally.

To find the equivalent fraction and create the same size units, the areas are decomposed horizontally and vertically, respectively. Now, the unit fractions are the same in each model or equation, and students can easily compare. The topic culminates with students comparing pairs of fractions and, by doing so, deciding which strategy is either necessary or efficient: reasoning using benchmarks and what they know about units, drawing a model (such as a number line, a tape diagram, or an area model), or the general method of finding like denominators through multiplication.

$$\frac{2}{3} < \frac{3}{4}$$



$\frac{2}{3} = \frac{8}{12}$ $\frac{3}{4} = \frac{9}{12}$

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 12 - 13

Objective: Reason using benchmarks to compare two fractions on the number line.

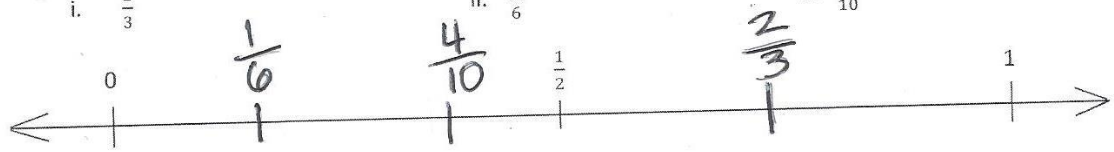
Homework Key (12)

1. a. Points plotted appropriately for $\frac{2}{3}, \frac{1}{6}, \frac{4}{10}$
   b. i. >
      ii. >
2. a. Points plotted appropriately for $\frac{5}{12}, \frac{3}{4}, \frac{2}{6}$
   b. Answers will vary.
   c. Explanations will vary.
3. a. >; explanations will vary.
   b. >; explanations will vary.
   c. >; explanations will vary.
   d. <; explanations will vary.
   e. >; explanations will vary.
   f. >; explanations will vary.
   g. <; explanations will vary.
   h. >; explanations will vary.
   i. >; explanations will vary.
   j. >; explanations will vary.

Homework Samples (12)

1\.
a. Plot the following points on the number line without measuring.

i. $\frac{2}{3}$ ii. $\frac{1}{6}$ iii. $\frac{4}{10}$



b. Use the number line in Part (a) to compare the fractions by writing >, <, or = on the lines.

i. $\frac{2}{3}$ > $\frac{1}{2}$ ii. $\frac{4}{10}$ > $\frac{1}{6}$

3\. Compare the fractions given below by writing > or < on the lines.
Give a brief explanation for each answer referring to the benchmark of 0, $\frac{1}{2}$, and 1.

Use benchmarks of 0, $\frac{1}{2}$ and 1 to help.

a. $\frac{1}{2}$ > $\frac{1}{4}$

$\frac{1}{2}$ is bigger than $\frac{1}{4}$.

b. $\frac{6}{8}$ > $\frac{1}{2}$

$\frac{6}{8}$ is closer to 1.

Lesson 13

Homework Key

1. Points plotted appropriately for $\frac{3}{2}, \frac{9}{5}, \frac{14}{10}$
2. a. <

   b. <
3. Points plotted appropriately for $\frac{12}{9}, \frac{6}{5}, \frac{18}{15}$
4. Explanations will vary.
5. a. <; explanations will vary.

   b. <; explanations will vary.

   c. >; explanations will vary.

   d. <; explanations will vary

   e. <; explanations will vary.

   f. <; explanations will vary.

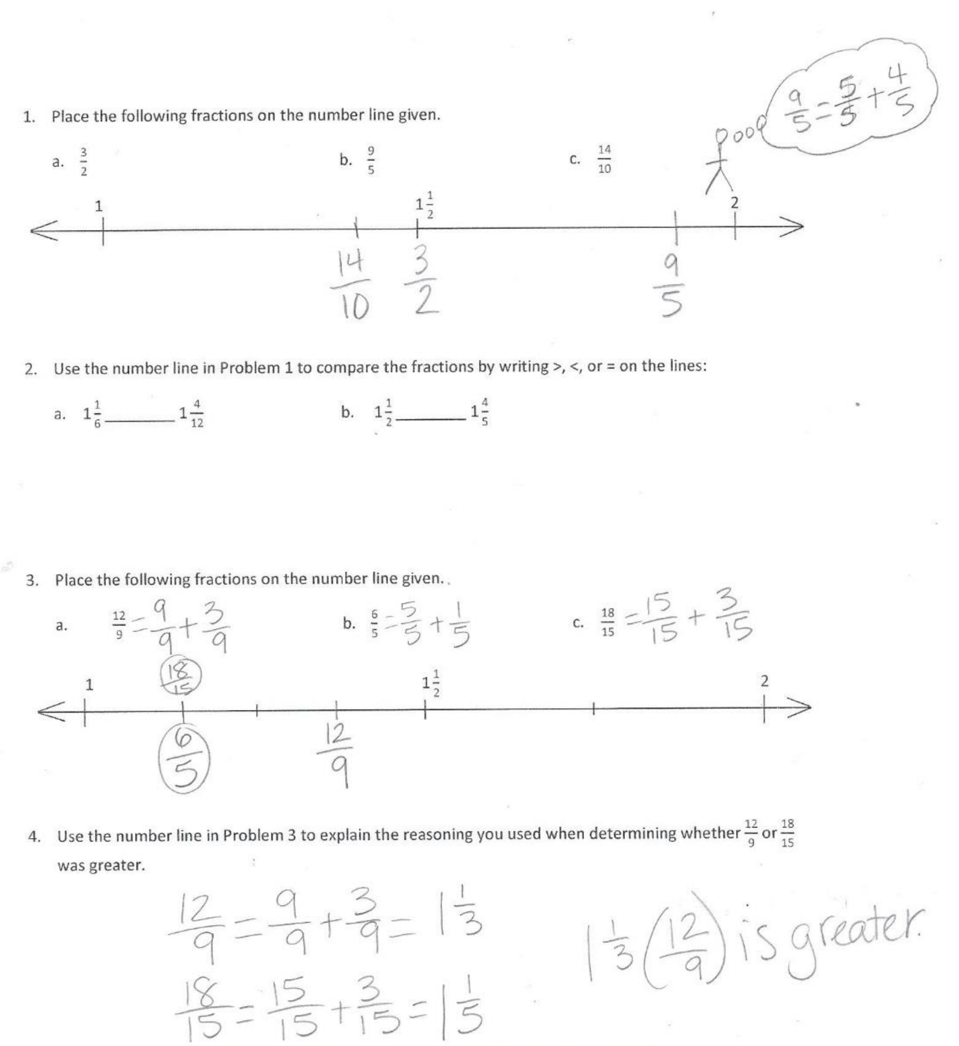
   g. <; explanations will vary.

   h. <; explanations will vary.

   i. <; explanations will vary.

   j. >; explanations will vary.

Homework Samples



1. Place the following fractions on the number line given.

a. $\frac{3}{2}$ b. $\frac{9}{5}$ c. $\frac{14}{10}$

$\frac{9}{5} = \frac{5}{5} + \frac{4}{5}$

2. Use the number line in Problem 1 to compare the fractions by writing >, <, or = on the lines:

a. $1\frac{1}{6}$ ________ $1\frac{4}{12}$ b. $1\frac{1}{2}$ ________ $1\frac{4}{5}$

3. Place the following fractions on the number line given.

a. $\frac{12}{9} = \frac{9}{9} + \frac{3}{9}$ b. $\frac{6}{5} = \frac{5}{5} + \frac{1}{5}$ c. $\frac{18}{15} = \frac{15}{15} + \frac{3}{15}$

4. Use the number line in Problem 3 to explain the reasoning you used when determining whether $\frac{12}{9}$ or $\frac{18}{15}$ was greater.

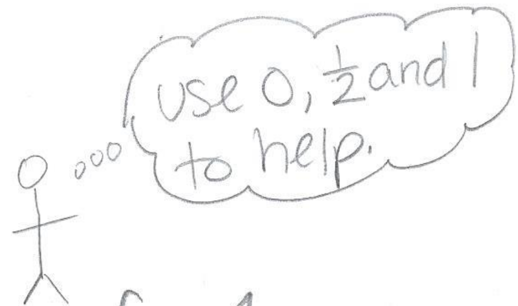
$\frac{12}{9} = \frac{9}{9} + \frac{3}{9} = 1\frac{1}{3}$

$\frac{18}{15} = \frac{15}{15} + \frac{3}{15} = 1\frac{1}{5}$

$1\frac{1}{3}$ $\left(\frac{12}{9}\right)$ is greater.

Lesson 13 (continued)

5. Compare the fractions given below by writing > or < on the lines.
   Give a brief explanation for each answer referring to benchmarks.



a. $\frac{2}{5}$ < $\frac{6}{8}$

$\frac{6}{8}$ is closer to 1.

b. $\frac{6}{10}$ < $\frac{5}{6}$

$\frac{5}{6}$ is $\frac{1}{6}$ away from 1.
$\frac{6}{10}$ is $\frac{4}{10}$ away from 1.

Lesson 14 - 15

Objective: Find common units or number of units to compare two fractions.

Homework Key (14)

1. a. >
   b. >
   c. >
   d. >
2. a. >; explanations will vary.
   b. Answer provided.
   c. >; explanations will vary.
   d. <; explanations will vary.
3. a. Tape diagrams model $\frac{3}{4} > \frac{7}{12}$
   b. Tape diagrams model $\frac{2}{4} > \frac{1}{8}$
   c. Tape diagrams model $1\frac{4}{10} < 1\frac{3}{5}$
4. a. Number line models fractions; >
   b. Number line models fractions; >
   c. Number line models fractions; >
   d. Number line models fractions; >
5. a. <
   b. <
   c. >
   d. >
   e. =
   f. <
   g. >
   h. >
6. Simon; picture supports answer

Lesson 14 (continued)

Homework Samples

1. Compare the pairs of fractions by reasoning about the size of the units. Use >, <, or =.

   a. 1 third __>__ 1 sixth

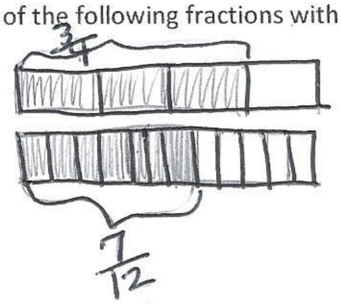
   b. 2 halves __>__ 2 thirds

   c. 2 fourths __>__ 2 sixths

   d. 5 eighths __>__ 5 tenths

3. Draw two tape diagrams to model each pair of the following fractions with related denominators. Use >, <, or = to compare.

   a. $\frac{3}{4}$ __>__ $\frac{7}{12}$



Lesson 15

Homework Key

1. a. Area models prove $\frac{1}{2} < \frac{3}{5}$
   b. Area models prove $\frac{2}{3} < \frac{3}{4}$
   c. Area models prove $\frac{4}{6} > \frac{5}{8}$
   d. Area models prove $\frac{2}{7} < \frac{3}{5}$
   e. Area models prove $\frac{4}{6} = \frac{6}{9}$
   f. Area models prove $\frac{4}{5} < \frac{10}{12}$
2. a. >
   b. >
   c. >
   d. >
3. a. >
   b. <
   c. =
   d. <
4. Explanations will vary.

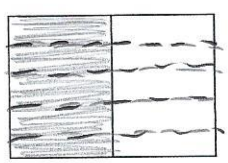
Homework Samples

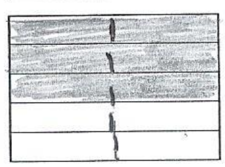
1. Draw an area model for each pair of fractions, and use it to compare the two fractions by writing >, <, or = on the line. The first two have been partially done for you. Each rectangle represents 1.

a. $\frac{1}{2}$ ___<___ $\frac{3}{5}$

$\frac{1\times5}{2\times5} = \frac{5}{10}$ $\quad \frac{3\times2}{5\times2} = \frac{6}{10}$

$\frac{5}{10} < \frac{6}{10}$ so $\frac{1}{2} < \frac{3}{5}$



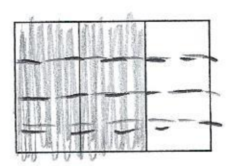


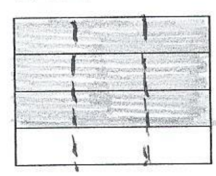
b. $\frac{2}{3}$ ___<___ $\frac{3}{4}$

$\frac{2\times4}{3\times4} = \frac{8}{12}$

$\frac{3\times3}{4\times3} = \frac{9}{12}$

$\frac{8}{12} < \frac{9}{12}$ so $\frac{2}{3} < \frac{3}{4}$





Lesson 15 (continued)

2. Rename the fractions, as needed, using multiplication in order to compare each pair of fractions by writing >, <, or =.

a. $\frac{2}{3}$ ___>___ $\frac{2}{4}$

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ $\frac{2 \times 3}{4 \times 3} = \frac{6}{12}$

b. $\frac{4}{7}$ ______ $\frac{1}{2}$

c. $\frac{5}{4}$ ______ $\frac{9}{8}$

d. $\frac{8}{12}$ ______ $\frac{5}{8}$

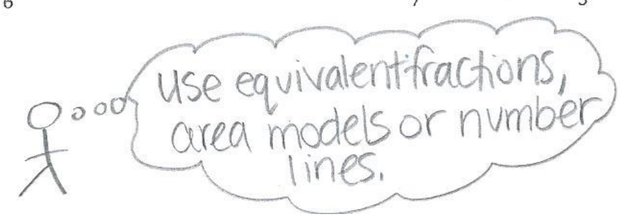
3. Use any method to compare the fractions. Record your answer using >, <, or =.

a. $\frac{8}{9}$ ______ $\frac{2}{3}$

b. $\frac{4}{7}$ ______ $\frac{4}{5}$

c. $\frac{3}{2}$ ______ $\frac{9}{6}$

d. $\frac{11}{7}$ ______ $\frac{5}{3}$



Grade 4 Module 5 Topic D

Fraction Addition and Subtraction

Focus Standard:

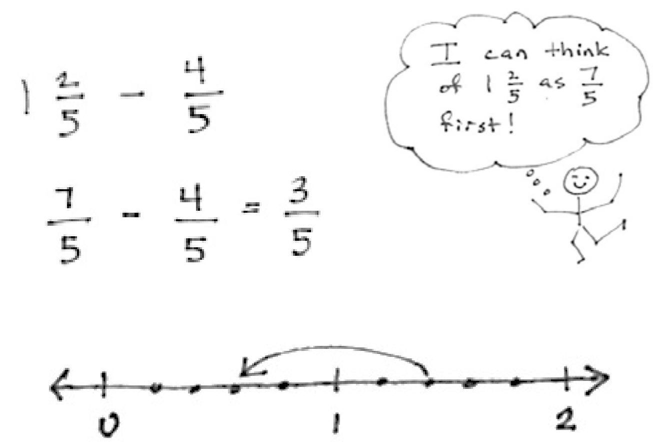
4.NF.3ad Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.

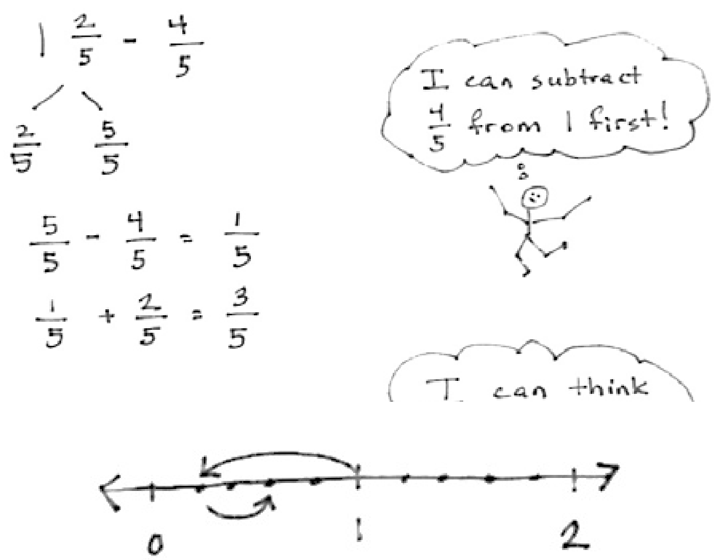
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Instructional Days Recommended: 6

Topic D bridges students' understanding of whole number addition and subtraction to fractions. Everything that they know to be true of addition and subtraction with whole numbers now applies to fractions. Addition is finding a total by combining like units. Subtraction is finding an unknown part. Implicit in the equations 3 + 2 = 5 and 2 = 5 – 3 is the assumption that the numbers are referring to the *same* units.

In Lessons 16 and 17, students generalize familiar facts about whole number addition and subtraction to work with fractions. Just as 3 apples – 2 apples = 1 apple, students note that 3 fourths – 2 fourths = 1 fourth. Just as 6 days + 3 days = 9 days = 1 week 2 days, students note that $\frac{6}{7} + \frac{3}{7} = \frac{9}{7} = \frac{7}{7} + \frac{2}{7} = 1\frac{2}{7}$. In Lesson 17, students decompose a whole into a fraction having the same denominator as the subtrahend. For example, 1 – 4 fifths becomes 5 fifths – 4 fifths = 1 fifth, connecting with Topic B skills. They then see that, when solving $1\frac{2}{5} - \frac{4}{5}$, they have a choice of subtracting $\frac{4}{5}$ from $\frac{7}{5}$ or from 1 (as pictured to the right). Students model with tape diagrams and number lines to understand and then verify their numerical work.



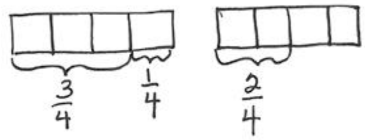


In Lesson 18, students add more than two fractions and see sums of more than one whole, such as $\frac{2}{8} + \frac{5}{8} + \frac{7}{8} = \frac{14}{8}$. As students move into problem solving in Lesson 19, they create tape diagrams or number lines to represent and solve fraction addition and subtraction word problems (see example below). These problems bridge students into work with mixed numbers, which follows the Mid-Module Assessment.

Mary mixed $\frac{3}{4}$ cup of wheat flour, $\frac{2}{4}$ cup of rice flour, and $\frac{1}{4}$ cup of oat flour for her bread dough. How many cups of flour did she put in her bread in all?

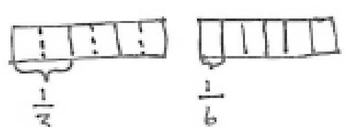
$$\frac{3}{4}+\frac{2}{4}+\frac{1}{4}=\frac{6}{4}$$

$$\frac{6}{4}=\frac{4}{4}+\frac{2}{4}=1+\frac{2}{4}=1\frac{2}{4}$$

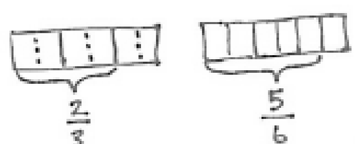


Mary used $\frac{6}{4}$ or $1\frac{2}{4}$ cups flour.

In Lessons 20 and 21, students add fractions with related units, where one denominator is a multiple (or factor) of the other. To add such fractions, a decomposition is necessary. Decomposing one unit into another is familiar territory: Students have had ample practice composing and decomposing in Topics A and B when working with place value units, converting units of measurement, and using the distributive property. For example, they have converted between equivalent measurement units (e.g., 100 cm = 1 m), and they have used such conversions to do arithmetic (e.g., 1 meter – 54 centimeters). With fractions, the concept is the same. To find the sum of $\frac{1}{2}$ and $\frac{1}{4}$, one simply renames (converts, decomposes) $\frac{1}{2}$ as $\frac{2}{4}$, and adds $\frac{1}{2}+\frac{2}{4}$, = $\frac{3}{4}$,. All numerical work is accompanied by visual models that allow students to use and apply their known skills and understandings. The addition of fractions with related units is also foundational to decimal work when adding tenths and hundredths in Module 6. Please note that addition of fractions with related denominators will not be assessed.



$$\frac{2}{6}+\frac{1}{6}=\frac{3}{6}$$



$$\frac{4}{6}+\frac{5}{6}=\frac{9}{6}=1\frac{3}{6}$$

$\frac{6}{6}$ $\frac{3}{6}$

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 16

Objective: Use visual models to add and subtract two fractions with the same units.

Homework Key

1. a. 1 sixth
   b. 2 tenths
   c. 1 fourth
   d. 3 thirds
2. a. $\frac{1}{5}$
   b. $\frac{4}{9}$
   c. $\frac{4}{12}$
   d. $\frac{2}{6}$
   e. $\frac{3}{3}$
   f. $\frac{2}{4}$
3. a. Answer provided
   b. Number bond shows $\frac{11}{8}$ is $\frac{8}{8}$ and $\frac{3}{8}$; $1\frac{3}{8}$
   c. Number bond shows $\frac{6}{5}$ is $\frac{5}{5}$ and $\frac{1}{5}$; $1\frac{1}{5}$
   d. Number bond shows $\frac{5}{4}$ is $\frac{4}{4}$ and $\frac{1}{4}$; $1\frac{1}{4}$
   e. Number bond shows $\frac{8}{7}$ is $\frac{7}{7}$ and $\frac{1}{7}$; $1\frac{1}{7}$
   f. Number bond shows $\frac{12}{10}$ is $\frac{10}{10}$ and $\frac{2}{10}$; $1\frac{2}{10}$
4. a. 6 fifths
   b. 7 eighths
5. a. $\frac{9}{11}$
   b. $\frac{9}{10}$
6. a. Number bond shows $\frac{6}{4}$ is $\frac{4}{4}$ and $\frac{2}{4}$; $1\frac{2}{4}$
   b. Number bond shows $\frac{14}{12}$ is $\frac{12}{12}$ and $\frac{2}{12}$; $1\frac{2}{12}$
   c. Number bond shows $\frac{12}{8}$ is $\frac{8}{8}$ and $\frac{4}{8}$; $1\frac{4}{8}$
   d. Number bond shows $\frac{13}{10}$ is $\frac{10}{10}$ and $\frac{3}{10}$; $1\frac{3}{10}$
   e. Number bond shows $\frac{9}{5}$ is $\frac{5}{5}$ and $\frac{4}{5}$; $1\frac{4}{5}$
   f. Number bond shows $\frac{6}{3}$ is $\frac{3}{3}$ and $\frac{3}{3}$; 2
7. a. Number line accurately models $\frac{11}{9} - \frac{5}{9} = \frac{6}{9}$
   b. Number line accurately models $\frac{13}{12} + \frac{4}{12} = \frac{17}{12}$

Homework Samples

1. Solve.

   a. 3 sixths – 2 sixths = 1 sixth $\left(\frac{1}{6}\right)$

   b. 5 tenths – 3 tenths = 2 tenths $\left(\frac{2}{10}\right)$

6. Solve. Use a number bond to decompose the sum. Record your final answer as a mixed number.

   a. $\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = 1\frac{2}{4}$ $\frac{6}{4}$ $\frac{4}{4}$ $\frac{2}{4}$

   b. $\frac{8}{12} + \frac{6}{12}$

   c. $\frac{5}{8} + \frac{7}{8}$

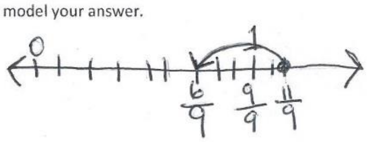
   d. $\frac{8}{10} + \frac{5}{10}$

   e. $\frac{3}{5} + \frac{6}{5}$

   f. $\frac{4}{3} + \frac{2}{3}$

7. Solve. Use a number line to model your answer.

   a. $\frac{11}{9} - \frac{5}{9} = \frac{6}{9}$



Lesson 17

Objective: Use visual models to add and subtract two fractions with the same units, including subtracting from one whole.

Homework Key

1. a. $\frac{5}{6}+\frac{4}{6}=\frac{9}{6}$, $\frac{4}{6}+\frac{5}{6}=\frac{9}{6}$, $\frac{9}{6}-\frac{5}{6}=\frac{4}{6}$, $\frac{9}{6}-\frac{4}{6}=\frac{5}{6}$
   b. $\frac{5}{9}+\frac{8}{9}=\frac{13}{9}$, $\frac{8}{9}+\frac{5}{9}=\frac{13}{9}$, $\frac{13}{9}-\frac{5}{9}=\frac{8}{9}$, $\frac{13}{9}-\frac{8}{9}=\frac{5}{9}$
2. a. $\frac{3}{8}$; number line models solution; solved by counting up and subtracting
   b. $\frac{3}{5}$; number line models solution; solved by counting up and subtracting
   c. $\frac{4}{6}$; number line models solution; solved by counting up and subtracting
   d. $\frac{3}{4}$; number line models solution; solved by counting up and subtracting
   e. $\frac{2}{3}$; number line models solution; solved by counting up and subtracting
   f. $\frac{4}{5}$; Number line models solution; solved by counting up and subtracting
3. a. Answer provided
   b. $\frac{8}{8}+\frac{3}{8}=\frac{11}{8}$, $\frac{11}{8}-\frac{7}{8}=\frac{4}{8}$; $\frac{8}{8}-\frac{7}{8}=\frac{1}{8}$, $\frac{1}{8}+\frac{3}{8}=\frac{4}{8}$; number bond shows $1\frac{3}{8}$ is $\frac{8}{8}$ and $\frac{3}{8}$
   c. $\frac{4}{4}+\frac{1}{4}=\frac{5}{4}$, $\frac{5}{4}-\frac{3}{4}=\frac{2}{4}$; $\frac{4}{4}-\frac{3}{4}=\frac{1}{4}$, $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$; number bond shows $1\frac{1}{4}$ is $\frac{4}{4}$ and $\frac{1}{4}$
   d. $\frac{7}{7}+\frac{2}{7}=\frac{9}{7}$, $\frac{9}{7}-\frac{5}{7}=\frac{4}{7}$; $\frac{7}{7}-\frac{5}{7}=\frac{2}{7}$, $\frac{2}{7}+\frac{2}{7}=\frac{4}{7}$; number bond shows $1\frac{2}{7}$ is $\frac{7}{7}$ and $\frac{2}{7}$
   e. $\frac{10}{10}+\frac{3}{10}=\frac{13}{10}$, $\frac{13}{10}-\frac{7}{10}=\frac{6}{10}$; $\frac{10}{10}-\frac{7}{10}=\frac{3}{10}$, $\frac{3}{10}+\frac{3}{10}=\frac{6}{10}$; number bond shows $1\frac{3}{10}$ is $\frac{10}{10}$ and $\frac{3}{10}$

Homework Sample

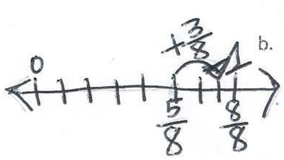
1. Use the following three fractions to write two subtraction and two addition number sentences.

a. $\frac{5}{6}$, $\frac{4}{6}$, $\frac{9}{6}$	b. $\frac{5}{9}$, $\frac{13}{9}$, $\frac{8}{9}$
$\frac{5}{6}+\frac{4}{6}=\frac{9}{6}$ $\frac{4}{6}+\frac{5}{6}=\frac{9}{6}$ $\frac{9}{6}-\frac{5}{6}=\frac{4}{6}$ $\frac{9}{6}-\frac{4}{6}=\frac{5}{6}$	

2. Solve. Model each subtraction problem with a number line, and solve by both counting up and subtracting.

a. $1-\frac{5}{8}$

$\frac{8}{8}-\frac{5}{8}=\frac{3}{8}$



b. $1-\frac{2}{5}$

Lesson 18

Objective: Add and subtract more than two fractions.

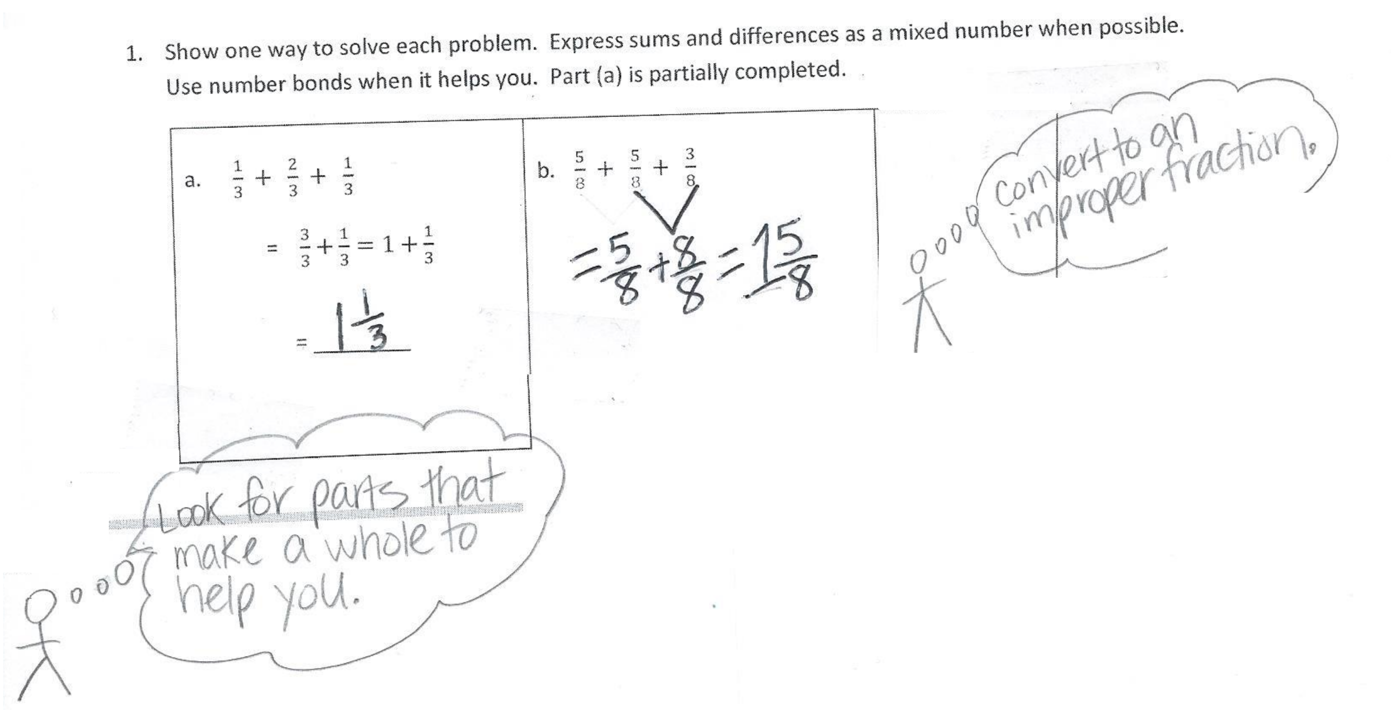
Homework Key

1. a. $1\frac{1}{3}$
   b. $1\frac{5}{8}$
   c. $1\frac{5}{6}$
   d. $\frac{11}{12}$
   e. $1\frac{3}{7}$
   f. 2
   g. $\frac{6}{10}$
   h. $\frac{3}{5}$
   i. 2

2. Answers will vary.

3. Answers will vary.

Homework Sample


1. Show one way to solve each problem. Express sums and differences as a mixed number when possible. Use number bonds when it helps you. Part (a) is partially completed.

a. $\frac{1}{3} + \frac{2}{3} + \frac{1}{3}$

$= \frac{3}{3} + \frac{1}{3} = 1 + \frac{1}{3}$

$= 1\frac{1}{3}$

b. $\frac{5}{8} + \frac{5}{8} + \frac{3}{8}$

$= \frac{5}{8} + \frac{8}{8} = 1\frac{5}{8}$

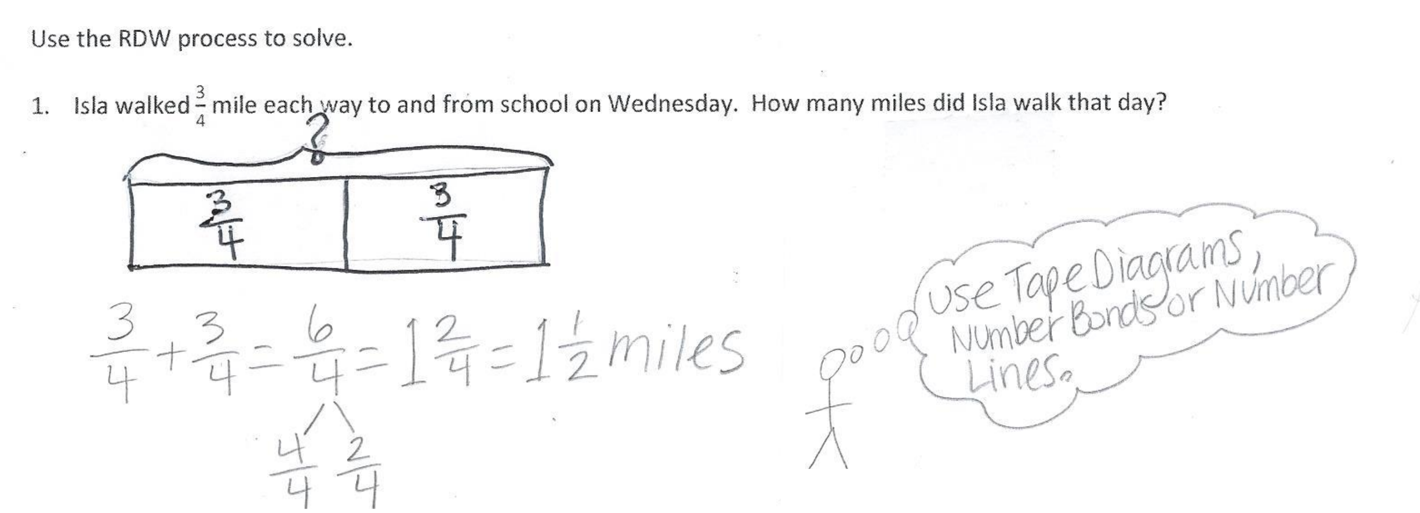
Lesson 19

Objective: Solve word problems involving addition and subtraction of fractions.

Homework Key

1. $1\frac{2}{4}$ mi
2. $\frac{2}{3}$ hr
3. $1\frac{7}{8}$ lb
4. $1\frac{7}{8}$ c
5. $\frac{1}{6}$
6. $\frac{2}{4}$ page

Homework Sample


Use the RDW process to solve.

1. Isla walked $\frac{3}{4}$ mile each way to and from school on Wednesday. How many miles did Isla walk that day?

Lesson 20 - 21

Objective: Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12.

Homework Key (20)

1. a. Tape diagrams model $\frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{1}{6}=\frac{3}{6}$
   b. Tape diagrams model $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$
   c. Tape diagrams model $\frac{3}{4}+\frac{1}{8}=\frac{6}{8}+\frac{1}{8}=\frac{7}{8}$
   d. Tape diagrams model
   $\frac{1}{4}+\frac{5}{12}=\frac{3}{12}+\frac{5}{12}=\frac{8}{12}$
   e. Tape diagrams model $\frac{3}{8}+\frac{1}{2}=\frac{3}{8}+\frac{4}{8}=\frac{7}{8}$
   f. Tape diagrams model
   $\frac{3}{5}+\frac{3}{10}=\frac{6}{10}+\frac{3}{10}=\frac{9}{10}$

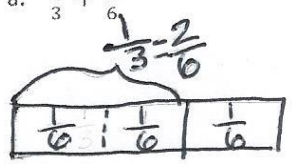
2. a. Answer provided
   b. Number line models $\frac{3}{5}+\frac{7}{10}$; $\frac{6}{10}+\frac{7}{10}=\frac{13}{10}$
   c. Number line models $\frac{5}{12}+\frac{1}{4}$; $\frac{5}{12}+\frac{3}{12}=\frac{8}{12}$
   d. Number line models $\frac{3}{4}+\frac{5}{8}$; $\frac{6}{8}+\frac{5}{8}=\frac{11}{8}$
   e. Number line models $\frac{7}{8}+\frac{3}{4}$; $\frac{7}{8}+\frac{6}{8}=\frac{13}{8}$
   f. Number line models $\frac{1}{6}+\frac{5}{3}$; $\frac{1}{6}+\frac{10}{6}=\frac{11}{6}$

3. $\frac{7}{6}$

Homework Sample

1. Use a tape diagram to represent each addend. Decompose one of the tape diagrams to make like units. Then, write the complete number sentence.

   a. $\frac{1}{3}+\frac{1}{6}$



$\frac{2}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$

Lesson 21

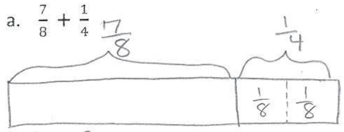
Homework Key

1. a. Tape diagrams represent $\frac{7}{8}$ and $\frac{2}{8}$; $\frac{7}{8}+\frac{2}{8}=\frac{9}{8}$; number bond shows $\frac{9}{8}$ as $\frac{8}{8}$ and $\frac{1}{8}$; $1\frac{1}{8}$
   b. Tape diagrams represent $\frac{4}{8}$ and $\frac{4}{8}$; $\frac{4}{8}+\frac{4}{8}=\frac{8}{8}$; 1
   c. Tape diagrams represent $\frac{4}{6}$ and $\frac{3}{6}$; $\frac{4}{6}+\frac{3}{6}=\frac{7}{6}$; number bond shows $\frac{7}{6}$ as $\frac{6}{6}$ and $\frac{1}{6}$; $1\frac{1}{6}$
   d. Tape diagrams represent $\frac{6}{10}$ and $\frac{8}{10}$; $\frac{6}{10}+\frac{8}{10}=\frac{14}{10}$; number bond shows $\frac{14}{10}$ as $\frac{10}{10}$ and $\frac{4}{10}$; $1\frac{4}{10}$
2. a. Number line models $\frac{4}{8}+\frac{5}{8}$; $\frac{4}{8}+\frac{5}{8}=\frac{9}{8}$; number bond shows $\frac{9}{8}$ as $\frac{8}{8}$ and $\frac{1}{8}$; $1\frac{1}{8}$
   b. Number line models $\frac{6}{8}+\frac{3}{8}$; $\frac{6}{8}+\frac{3}{8}=\frac{9}{8}$; number bond shows $\frac{9}{8}$ as $\frac{8}{8}$ and $\frac{1}{8}$; $1\frac{1}{8}$
   c. Number line models $\frac{4}{10}+\frac{8}{10}$; $\frac{4}{10}+\frac{8}{10}=\frac{12}{10}$; number bond shows $\frac{12}{10}$ as $\frac{10}{10}$ and $\frac{2}{10}$; $1\frac{2}{10}$
   d. Number line models $\frac{2}{6}+\frac{5}{6}$; $\frac{2}{6}+\frac{5}{6}=\frac{7}{6}$; number bond shows $\frac{7}{6}$ as $\frac{6}{6}$ and $\frac{1}{6}$; $1\frac{1}{6}$
3. a. $\frac{4}{8}+\frac{6}{8}=\frac{10}{8}=1\frac{2}{8}$
   b. $\frac{7}{8}+\frac{6}{8}=\frac{13}{8}=1\frac{5}{8}$
   c. $\frac{5}{6}+\frac{2}{6}=\frac{7}{6}=1\frac{1}{6}$
   d. $\frac{9}{10}+\frac{4}{10}=\frac{13}{10}=1\frac{3}{10}$
   e. $\frac{4}{12}+\frac{9}{12}=\frac{13}{12}=1\frac{1}{12}$
   f. $\frac{3}{6}+\frac{5}{6}=\frac{8}{6}=1\frac{2}{8}$
   g. $\frac{3}{12}+\frac{10}{12}=\frac{13}{12}=1\frac{1}{12}$
   h. $\frac{7}{10}+\frac{8}{10}=\frac{15}{10}=1\frac{5}{10}$

Homework Samples

1. Draw a tape diagram to represent each addend. Decompose one of the tape diagrams to make like units. Then, write a complete number sentence. Use a number bond to write each sum as a mixed number.

a. $\frac{7}{8}+\frac{1}{4}$



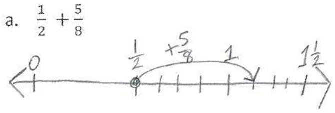
$\frac{7}{8}+\frac{2}{8}=\frac{9}{8}=1\frac{1}{8}$

$\frac{8}{8}$ $\frac{1}{8}$

2. Draw a number line to model the addition. Then, write a complete number sentence. Use a number bond to write each sum as a mixed number.

a. $\frac{1}{2}+\frac{5}{8}$

b. $\frac{3}{4}+\frac{3}{8}$



$\frac{1}{2}+\frac{5}{8}=\frac{4}{8}+\frac{5}{8}=\frac{9}{8}=1\frac{1}{8}$

$\frac{8}{8}$ $\frac{1}{8}$

Extending Fraction Equivalence to Fractions Greater Than 1

Focus Standards:

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

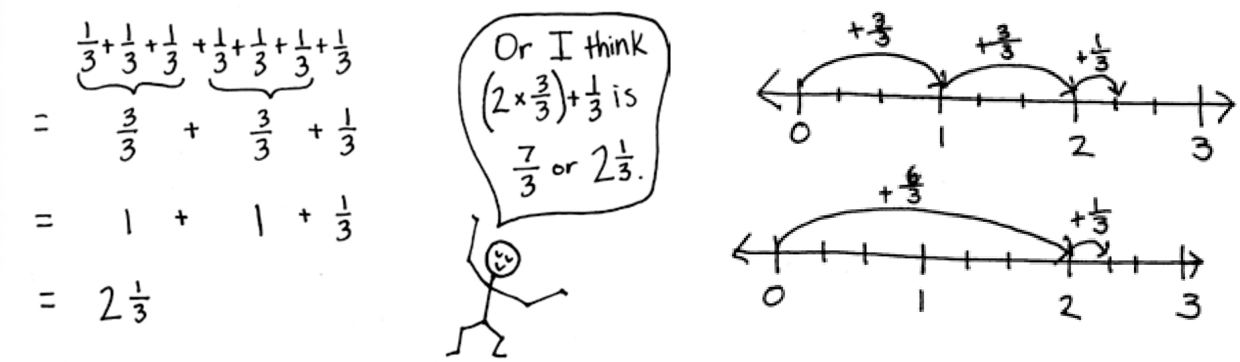
4.NF.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

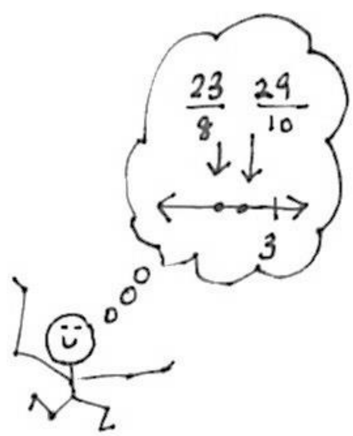
Instructional Days Recommended: 7

In Topic E, students study equivalence involving both ones and fractional units. In Lesson 22, they use decomposition and visual models to add and subtract fractions less than 1 to and from whole numbers, e.g., $4 + \frac{3}{4} = 4\frac{3}{4}$ and $4 - \frac{3}{4} = (3 + 1) - \frac{3}{4}$, subtracting the fraction from 1 using a number bond and a number line.
Lesson 23 has students use addition and multiplication to build fractions greater than 1 and then represent them on the number line. Fractions can be expressed both in mixed units of a whole number and a fraction or simply as a fraction, as pictured below, e.g., $7 \times \frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = \frac{7}{3} = 2\frac{1}{3}$.



In Lessons 24 and 25, students use decompositions to reason about the various equivalent forms in which a fraction greater than or equal to 1 may be presented, both as fractions and as mixed numbers. In Lesson 24, they decompose, for example, 11 fourths into 8 fourths and 3 fourths, $\frac{11}{4} = \frac{8}{4} + \frac{3}{4}$, or they can think of it as $\frac{11}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = 2 \times \frac{4}{4} + \frac{3}{4} = 2\frac{3}{4}$. In Lesson 25, students are then able to decompose the two wholes into 8 fourths so their original number can then be looked at as $\frac{8}{4} + \frac{3}{4}$ or $\frac{11}{4}$. In this way, they see that $2\frac{3}{4} = \frac{11}{4}$. This fact is further reinforced when they plot $\frac{11}{4}$ on the number line and see that it is at the same point as $2\frac{3}{4}$. Unfortunately, the term *improper fraction* carries some baggage. As many have observed, there is nothing *improper* about an improper fraction. Nevertheless, as a mathematical term, it is useful for describing a particular form in which a fraction may be presented (i.e., a fraction is improper if the numerator is greater than or equal to the denominator). Students do need practice in terms of converting between the various forms a fraction may take, but take care not to foster the misconception that every improper fraction *must* be converted to a mixed number.

Students compare fractions greater than 1 in Lessons 26 and 27. They begin by using their understanding of benchmarks to reason about which of two fractions is greater. This activity builds on students' rounding skills, having them identify the whole numbers and the halfway points between them on the number line. The relationship between the numerator and denominator of a fraction is a key concept here as students consider relationships to whole numbers, e.g., a student might reason that $\frac{23}{8}$ is less than $\frac{29}{10}$ because $\frac{23}{8}$ is 1 eighth less than 3, but $\frac{29}{10}$ is 1 tenth less than 3. They know each fraction is 1 fractional unit away from 3, and since $\frac{1}{8} > \frac{1}{10}$, then $\frac{23}{8} < \frac{29}{10}$. Students progress to finding and using like denominators to compare and order mixed numbers. Once again, students must use reasoning skills as they determine that, when they have two fractions with the same numerator, the larger fraction will have a larger unit (or smaller denominator). Conversely, when they have two fractions with the same denominator, the larger one will have the larger number of units (or larger numerator).



Lesson 28 concludes the topic with word problems requiring the interpretation of data presented in line plots. Students create line plots to display a given dataset that includes fraction and mixed number values. To do this, they apply their skill in comparing mixed numbers, both through reasoning and the use of common numerators or denominators. For example, a data set might contain both $1\frac{5}{9}$ and $\frac{14}{9}$, giving students the opportunity to determine that they must be plotted at the same point. They also use addition and subtraction to solve the problems.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 22

Objective: Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models.

Homework Key

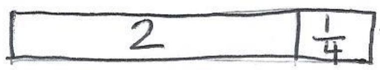
1. a. Tape diagram drawn; $2\frac{1}{4}$
   b. Tape diagram drawn; $3\frac{2}{3}$
   c. Tape diagram drawn; $1\frac{4}{5}$
   d. Tape diagram drawn; $2\frac{1}{4}$
2. a. $4\frac{5}{8}-\frac{5}{8}=4$, $4\frac{5}{8}-4=\frac{5}{8}$, $4+\frac{5}{8}=4\frac{5}{8}$, $\frac{5}{8}+4=4\frac{5}{8}$
   b. $6-\frac{2}{7}=5\frac{5}{7}$, $6-5\frac{5}{7}=\frac{2}{7}$, $5\frac{5}{7}+\frac{2}{7}=6$, $\frac{2}{7}+5\frac{5}{7}=6$
3. a. Answer provided
   b. $7\frac{1}{6}$; number bond shows 8 as 7 and $\frac{6}{6}$; number line drawn
   c. $6\frac{1}{5}$; number bond shows 7 as 6 and $\frac{5}{5}$; number line drawn
   d. $2\frac{7}{10}$; number bond shows 3 as 2 and $\frac{10}{10}$; number line drawn
4. a. $5\frac{3}{4}$; number bond shows 6 as 5 and $\frac{4}{4}$
   b. $6\frac{8}{10}$; number bond shows 7 as 6 and $\frac{10}{10}$
   c. $4\frac{1}{6}$; number bond shows 5 as 4 and $\frac{6}{6}$
   d. $5\frac{2}{8}$; number bond shows 6 as 5 and $\frac{8}{8}$
   e. $2\frac{1}{8}$; number bond shows 3 as 2 and $\frac{8}{8}$
   f. $25\frac{3}{10}$; number bond shows 26 as 25 and $\frac{10}{10}$

Lesson 22 (continued)

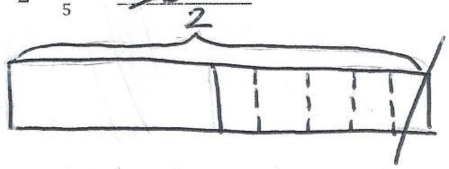
Homework Samples

1. Draw a tape diagram to match each number sentence. Then, complete the number sentence.

   a. $2 + \frac{1}{4} =$ $2\frac{1}{4}$



   c. $2 - \frac{1}{5} =$ $1\frac{4}{5}$



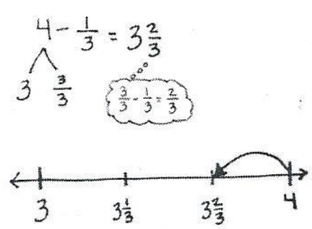
2. Use the following three numbers to write two subtraction and two addition number sentences.

   a. $4,\ 4\frac{5}{8},\ \frac{5}{8}$

   b. $\frac{2}{7},\ 5\frac{5}{7},\ 6$

3. Solve using a number bond. Draw a number line to represent each number sentence. The first one has been done for you.

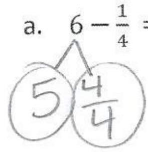
   a. $4 - \frac{1}{3} =$ $3\frac{2}{3}$



   b. $8 - \frac{5}{6} =$ ______

4. Complete the subtraction sentences using number bonds.

   a. $6 - \frac{1}{4} =$ $5\frac{3}{4}$



$5\frac{4}{4} - \frac{1}{4} = 5\frac{3}{4}$

   b. $7 - \frac{2}{10} =$ ______

Lesson 23

Objective: Add and multiply unit fractions to build fractions greater than 1 using visual models.

Homework Key

1. a. $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}$; $\frac{0}{4}, \frac{4}{4}$ circled; 0, 1 recorded
   b. $\frac{0}{6}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}, \frac{11}{6}, \frac{12}{6}, \frac{13}{6}, \frac{14}{6}$; $\frac{0}{6}, \frac{6}{6}, \frac{12}{6}$ circled ; 0, 1, 2 recorded
2. $\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)=4$
3. a. Answer provided
   b. $10 \times \frac{1}{2} = 5 \times \frac{2}{2} = 5$; number line supports answer.
   c. $8 \times \frac{1}{4} = 2 \times \frac{4}{4} = 2$; number line supports answer.
4. a. Answer provided
   b. $7 \times \frac{1}{4} = \left(1 \times \frac{4}{4}\right) + \frac{3}{4} = 1 + \frac{3}{4} = 1\frac{3}{4}$; number line supports answer.
   c. $11 \times \frac{1}{5} = \left(2 \times \frac{5}{5}\right) + \frac{1}{5} = 2 + \frac{1}{5} = 2\frac{1}{5}$; number line supports answer.
   d. $7 \times \frac{1}{2} = \left(3 \times \frac{2}{2}\right) + \frac{1}{2} = 3 + \frac{1}{2} = 3\frac{1}{2}$; number line supports answer.
   e. $9 \times \frac{1}{5} = \left(1 \times \frac{5}{5}\right) + \frac{4}{5} = 1 + \frac{4}{5} = 1\frac{4}{5}$; number line supports answer.

Homework Samples

1. Circle any fractions that are equivalent to a whole number. Record the whole number below the fraction.

   a. Count by 1 fourths. Start at 0 fourths. Stop at 6 fourths.

   $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}$

   0   1

   b. Count by 1 sixths. Start at 0 sixths. Stop at 14 sixths.

   $\frac{0}{6}, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}, \frac{11}{6}, \frac{12}{6}, \frac{13}{6}, \frac{14}{6}$

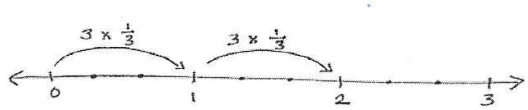
   0   1   2

2. Use parentheses to show how to make ones in the following number sentence.

$$\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)=4$$

3. Multiply, as shown below. Draw a number line to support your answer.

   a. $6 \times \frac{1}{3}$



$$6 \times \frac{1}{3} = 2 \times \frac{3}{3} = 2$$

Lesson 24 - 25

Objective: Decompose and compose fractions greater than 1 to express them in various forms.

Homework Key (24)

1. a. Answer provided.
   b. $3\frac{1}{4}$; number bond shows $\frac{13}{4}$ as 3 and $\frac{1}{4}$; number line drawn
   c. $3\frac{1}{5}$; number bond shows $\frac{16}{5}$ as 3 and $\frac{1}{5}$; number line drawn
   d. $7\frac{1}{2}$; number bond shows $\frac{15}{2}$ as 7 and $\frac{1}{2}$; number line drawn
   e. $5\frac{2}{3}$; number bond shows $\frac{17}{3}$ as 5 and $\frac{2}{3}$; number line drawn
2. a. Answer provided.
   b. $\frac{13}{2} = \frac{2 \times 6}{2} + \frac{1}{2} = 6 + \frac{1}{2} = 6\frac{1}{2}$; number line drawn
   c. $\frac{18}{4} = \frac{4 \times 4}{4} + \frac{2}{4} = 4 + \frac{2}{4} = 4\frac{2}{4}$; number line drawn
3. a. $4\frac{2}{3}$
   b. $4\frac{1}{4}$
   c. $5\frac{2}{5}$
   d. $4\frac{4}{6}$
   e. $3\frac{2}{7}$
   f. $4\frac{5}{8}$
   g. $5\frac{6}{9}$
   h. $7\frac{4}{10}$
   i. $3\frac{9}{12}$

Lesson 24 (continued)

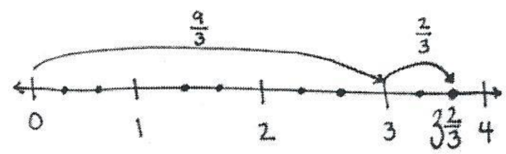
Homework Samples

1. Rename each fraction as a mixed number by decomposing it into two parts as shown below. Model the decomposition with a number line and a number bond.

a. $\frac{11}{3}$

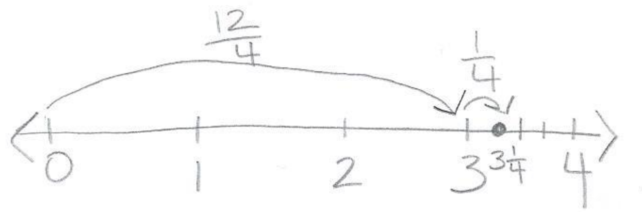
$\frac{11}{3} = \frac{9}{3} + \frac{2}{3} = 3 + \frac{2}{3} = 3\frac{2}{3}$

$\frac{9}{3}$ $\frac{2}{3}$



b. $\frac{13}{4} = \frac{12}{4} + \frac{1}{4} = 3 + \frac{1}{4} = 3\frac{1}{4}$

$\frac{12}{4}$ $\frac{1}{4}$



3. Convert each fraction to a mixed number.

a. $\frac{14}{3} = \frac{12}{3} + \frac{2}{3} = 4\frac{2}{3}$

Lesson 25

Homework Key

1. a. Answer provided.
   b. $\frac{22}{5}$; number line models work.
   c. $\frac{43}{8}$; number line models work.
   d. $\frac{37}{10}$; number line models work.
   e. $\frac{56}{9}$; number line models work.

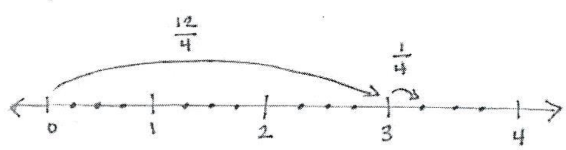
2. a. Answer provided.
   b. $5\frac{2}{3} = 5 + \frac{2}{3} = \left(5 \times \frac{3}{3}\right) + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$
   c. $4\frac{1}{5} = 4 + \frac{1}{5} = \left(4 \times \frac{5}{5}\right) + \frac{1}{5} = \frac{20}{5} + \frac{1}{5} = \frac{21}{5}$
   d. $3\frac{7}{8} = 3 + \frac{7}{8} = \left(3 \times \frac{8}{8}\right) + \frac{7}{8} = \frac{24}{8} + \frac{7}{8} = \frac{31}{8}$

3. a. $\frac{7}{3}$
   b. $\frac{11}{4}$
   c. $\frac{17}{5}$
   d. $\frac{19}{6}$
   e. $\frac{53}{12}$
   f. $\frac{22}{5}$
   g. $\frac{41}{10}$
   h. $\frac{26}{5}$
   i. $\frac{35}{6}$
   j. $\frac{25}{4}$
   k. $\frac{15}{2}$
   l. $\frac{95}{12}$

Homework Samples

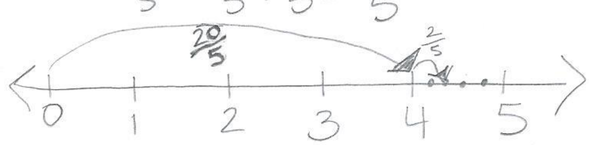
1. Convert each mixed number to a fraction greater than 1. Draw a number line to model your work.

a. $3\frac{1}{4}$



$3\frac{1}{4} = 3 + \frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}$

b. $4\frac{2}{5} = 4 + \frac{2}{5} = \frac{20}{5} + \frac{2}{5} = \frac{22}{5}$



3. Convert each mixed number to a fraction greater than 1.

a. $2\frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{7}{3}$

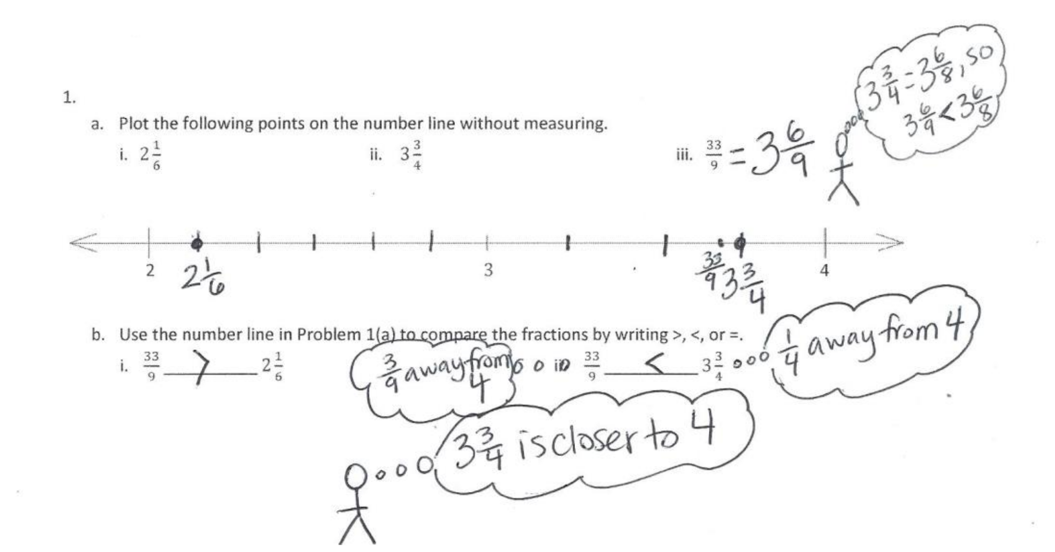
Lesson 26

Objective: Compare fractions greater than 1 by reasoning using benchmark fractions.

Homework Key

1. a. $2\frac{1}{6}$, $3\frac{3}{4}$, $\frac{33}{9}$ plotted
   b. i. >
      ii. <
2. a. $\frac{65}{8}$, $8\frac{5}{6}$, $\frac{29}{4}$ plotted
   b. i. >
      ii. <
   c. Explanations will vary.
3. a. <; explanations will vary.
   b. <; explanations will vary.
   c. <; explanations will vary.
   d. >; explanations will vary.
   e. >; explanations will vary.
   f. >; explanations will vary.
   g. <; explanations will vary.
   h. <; explanations will vary.
   i. <; explanations will vary.
   j. >; explanations will vary.

Homework Samples



1. a. Plot the following points on the number line without measuring.

i. $2\frac{1}{6}$ ii. $3\frac{3}{4}$ iii. $\frac{33}{9} = 3\frac{6}{9}$

$3\frac{3}{4} = 3\frac{6}{8}$, so $3\frac{6}{9} < 3\frac{6}{8}$

2 $2\frac{1}{6}$ 3 $\frac{33}{9}$ $3\frac{3}{4}$ 4

b. Use the number line in Problem 1(a) to compare the fractions by writing >, <, or =.

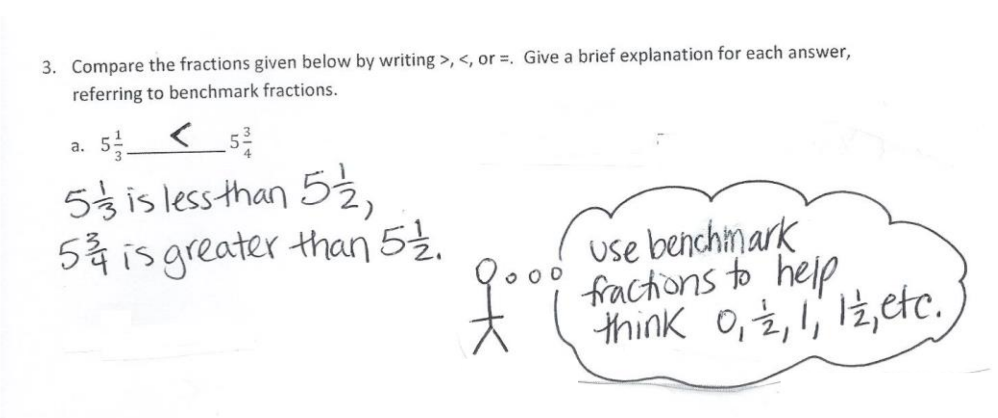
i. $\frac{33}{9}$ ___>___ $2\frac{1}{6}$

$\frac{3}{9}$ away from 4

ii. $\frac{33}{9}$ ___<___ $3\frac{3}{4}$

$\frac{1}{4}$ away from 4

$3\frac{3}{4}$ is closer to 4



3. Compare the fractions given below by writing >, <, or =. Give a brief explanation for each answer, referring to benchmark fractions.

a. $5\frac{1}{3}$ ___<___ $5\frac{3}{4}$

$5\frac{1}{3}$ is less than $5\frac{1}{2}$,
$5\frac{3}{4}$ is greater than $5\frac{1}{2}$.

use benchmark fractions to help think $0, \frac{1}{2}, 1, 1\frac{1}{2}$, etc.

Lesson 27

Objective: Compare fractions greater than 1 by creating common numerators or denominators.

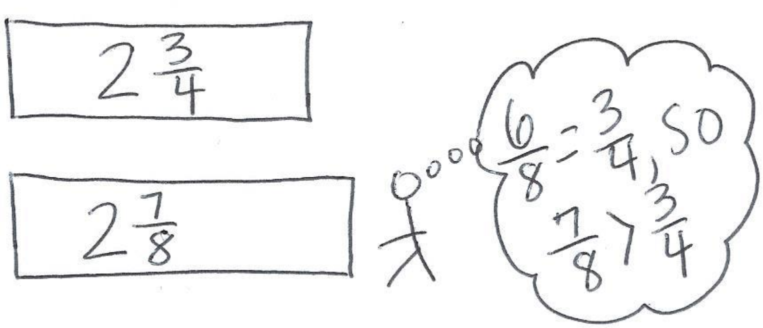
Homework Key

1. a. Tape diagram models comparison; <
   b. Tape diagram models comparison; =
   c. Tape diagram models comparison; >
   d. Tape diagram models comparison; <
2. a. Area model shows like units; >
   b. Area model shows like units; <
3. a. >
   b. <
   c. >
   d. <
   e. >
   f. <
   g. >
   h. >
   i. >
   j. >

Homework Samples

1. Draw a tape diagram to model each comparison. Use >, <, or = to compare.

a. $2\frac{3}{4}$ ___<___ $2\frac{7}{8}$ b. $10\frac{2}{6}$ ________ $10\frac{1}{3}$



3. Compare each pair of fractions using >, <, or = using any strategy.

a. $6\frac{1}{2}$ ___>___ $6\frac{3}{8}$

$6\frac{3}{8}$ is less than $6\frac{1}{2}$

$6\frac{4}{8}$ is equal to $6\frac{1}{2}$

Lesson 28

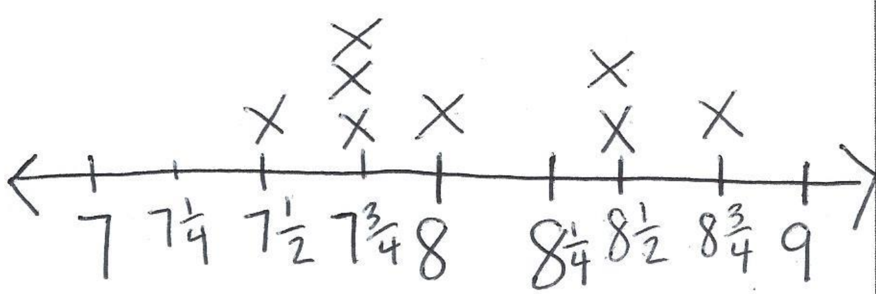
Objective: Solve word problems with line plots.

Homework Key

1. Line plot created accurately
2. a. Mary
   b. Ben
   c. 31 quarter inches
   d. $\frac{1}{4}$ inch
   e. $7\frac{1}{2} < 7\frac{3}{4}$
   f. 4
   g. 8
   h. $\frac{25}{2} > 8\frac{3}{4}$; Mr. Jones
3. Answers will vary.

Homework Sample

1. A group of children measured the lengths of their shoes. The measurements are shown in the table. Make a line plot to display the data.



Students	Length of Shoe (in inches)
Collin	$8\frac{1}{2}$
Dickon	$7\frac{3}{4}$
Ben	$7\frac{1}{2}$
Martha	$7\frac{3}{4}$
Lilias	8
Susan	$8\frac{1}{2}$
Frances	$7\frac{3}{4}$
Mary	$8\frac{3}{4}$

Grade 4 Module 5 Topic F

Addition and Subtraction of Fractions by Decomposition

Focus Standard:

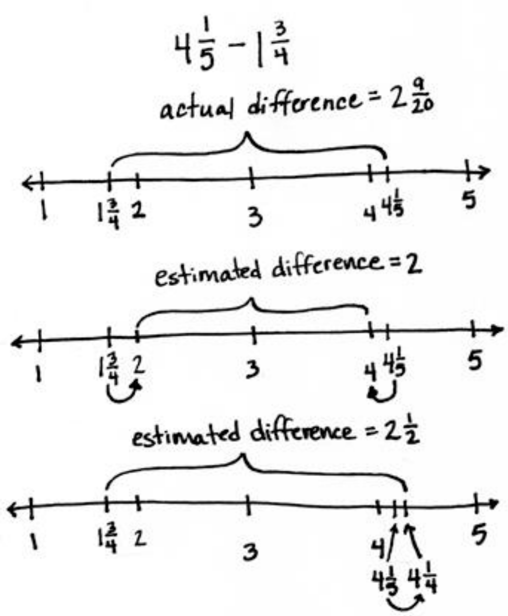
4.NF.3c Understand a fraction *a*/*b* with *a* > *1* as a sum of fractions 1/*b*.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

Instructional Days Recommended: 6

Topic F provides students with the opportunity to use their understandings of fraction addition and subtraction as they explore mixed number addition and subtraction by decomposition.

Lesson 29 focuses on the process of using benchmark numbers to estimate sums and differences of mixed numbers. Students once again call on their understanding of benchmark fractions as they determine, prior to performing the actual operation, what a reasonable outcome will be. One student might use benchmark whole numbers and reason, for example, that the difference between $4\frac{1}{5}$ and $1\frac{3}{4}$ is close to 2 because $4\frac{1}{5}$ is closer to 4 than 5, $1\frac{3}{4}$ is closer to 2 than 1, and the difference between 4 and 2 is 2. Another student might use familiar benchmark fractions and reason that the answer will be closer to $2\frac{1}{2}$ since $4\frac{1}{5}$ is about $\frac{1}{4}$ more than 4 and $1\frac{3}{4}$ is about $\frac{1}{4}$ less than 2, making the difference about a half more than 2 or $2\frac{1}{2}$.



In Lesson 30, students begin adding a mixed number to a fraction using unit form. They add like units, applying their Grades 1 and 2 understanding of completing a unit to add when the sum of the fractional units exceeds 1. Students ask, “How many more do we need to make one?” rather than “How many more do we need to make ten?” as was the case in Grade 1. A number bond decomposes the fraction to make one and can be modeled on the number line or using the arrow way, as shown to the right. Alternatively, a number bond can be used after adding like units, when the sum results in a mixed number with a fraction greater than 1, to decompose the fraction greater than 1 into ones and fractional units.

Directly applying what was learned in Lesson 30, Lesson 31 starts with adding like units, e.g., ones with ones and fourths with fourths, to add two mixed numbers. Students can, again, choose to make one before finding the sum or to decompose the sum to result in a proper mixed number.

$$5\frac{2}{4}+\frac{3}{4}=6\frac{1}{4}$$

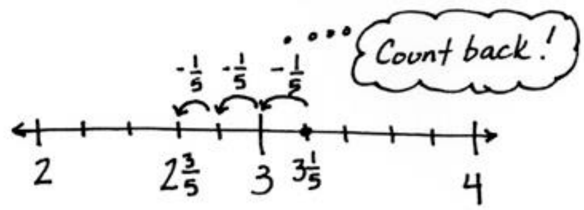
$$\frac{3}{4} \to \frac{2}{4},\ \frac{1}{4}$$

$$5\frac{2}{4}\xrightarrow{+\frac{2}{4}}6\xrightarrow{+\frac{1}{4}}6\frac{1}{4}$$

$$3\frac{9}{10}+2\frac{2}{10}$$
$$=5\frac{9}{10}+\frac{2}{10}$$
$$=5\frac{11}{10}$$
$$\frac{11}{10} \to \frac{10}{10},\ \frac{1}{10}$$
$$=6\frac{1}{10}$$

Lessons 32 and 33 follow the same sequence for subtraction. In Lesson 32, students simply subtract a fraction from a mixed number, using three main strategies both when there are and there are not enough fractional units. They count back or up, subtract from 1, or take one out to subtract from 1. In Lesson 33, students apply these strategies after subtracting the ones first. They model subtraction of mixed numbers using a number line or the arrow way.

$$3\frac{1}{5} - \frac{3}{5}$$



$$3\frac{1}{5} - \frac{3}{5} = 3 - \frac{2}{5} = 2\frac{3}{5}$$

$\frac{1}{5}$ $\frac{2}{5}$

Just like subtracting from one!

$$\frac{3}{5} \xrightarrow{+\frac{2}{5}} 1 \xrightarrow{+2} 3 \xrightarrow{+\frac{1}{5}} 3\frac{1}{5}$$

Count up!

$$\frac{2}{5} + 2 + \frac{1}{5} = 2\frac{3}{5}$$

$$3\frac{1}{5} - \frac{3}{5} = 2\frac{1}{5} + \frac{2}{5} = 2\frac{3}{5}$$

$2\frac{1}{5}$ 1

Take one out to subtract from one!

In Lesson 34, students learn another strategy for subtraction by decomposing the total into a whole number and a fraction greater than one to either subtract a fraction or a mixed number.

$$8\frac{1}{10} - \frac{8}{10} = 7\frac{11}{10} - \frac{8}{10} = 7\frac{3}{10}$$

7 $\frac{11}{10}$

$$11\frac{1}{5} - 2\frac{3}{5} = 9\frac{1}{5} - \frac{3}{5} = 8\frac{3}{5}$$

8 $\frac{6}{5}$

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 29

Objective: Estimate sums and differences using benchmark numbers.

Homework Key

1. a. 5; explanations will vary.
   b. 8; explanations will vary.
   c. 5; explanations will vary.
   d. 3; explanations will vary.
   e. 11; explanations will vary.
2. a. 7.5; explanations will vary.
   b. 2; explanations will vary.
   c. 10 or 10.5; explanations will vary.
3. Gina's; explanations will vary.
4. a. 24
   b. 26
   c. 7
   d. 4

Homework Sample

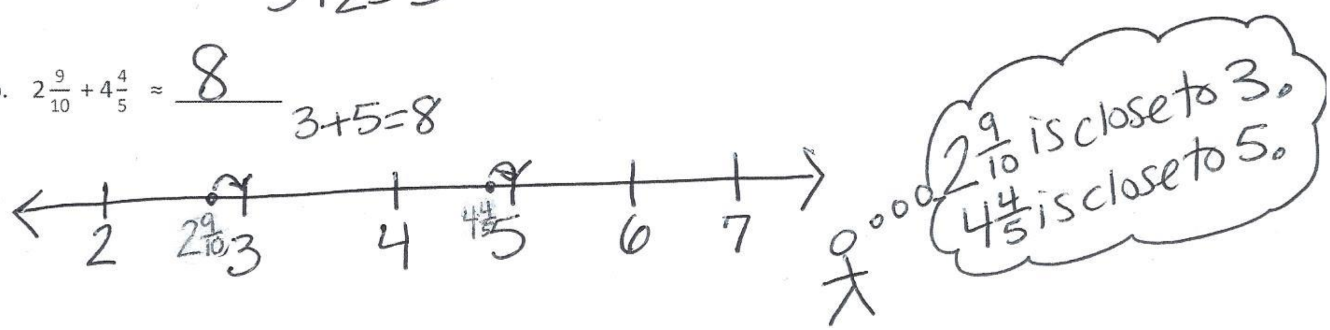
1. Estimate each sum or difference to the nearest half or whole number by rounding. Explain your estimate using words or a number line.

   a. $3\frac{1}{10} + 1\frac{3}{4} \approx$ 5

   $3\frac{1}{10}$ is close to 3. $1\frac{3}{4}$ is close to 2.

   $3+2=5$

   b. $2\frac{9}{10} + 4\frac{4}{5} \approx$ 8 $3+5=8$



Lesson 30

Objective: Add a mixed number and a fraction.

Homework Key

1. a. $4\frac{2}{3}$
   b. $5\frac{3}{4}$
   c. 4
   d. 8
2. a. $\frac{1}{6}$
   b. $\frac{4}{7}$
   c. $\frac{7}{8}$
   d. $\frac{8}{12}$
3. a. Number bond and arrow way used to make one; $3\frac{1}{5}$
   b. Number bond and arrow way used to make one; $4\frac{1}{3}$
   c. Number bond and arrow way used to make one; $5\frac{3}{6}$
4. a. $3\frac{1}{5}$
   b. $4\frac{2}{8}$
   c. $6\frac{1}{6}$
   d. $7\frac{3}{10}$
   e. $9\frac{4}{10}$
   f. $8\frac{7}{12}$
   g. $4\frac{48}{100}$
   h. $15\frac{39}{100}$
5. Explanations will vary.

Homework Samples

1. Solve.

a. $4\frac{1}{3} + \frac{1}{3} = 4\frac{2}{3}$

4 ones 1 third + 1 third

b. $5\frac{1}{4} + \frac{2}{4}$

c. $\frac{2}{6} + 3\frac{4}{6}$

d. $\frac{5}{8} + 7\frac{3}{8}$

2. Complete the number sentences.

a. $3\frac{5}{6} + \underline{\frac{1}{6}} = 4$ It's $\frac{1}{6}$ because $\frac{5}{6} + \frac{1}{6} = \frac{6}{6} = 1$	b. $5\frac{3}{7} + \_\_\_ = 6$
c. $5 = 4\frac{1}{8} + \_\_\_$	d. $15 = 14\frac{4}{12} + \_\_\_$

3. Draw a number bond and the arrow way to show how to make one. Solve.

a. $2\frac{4}{5} + \frac{2}{5}$

$\frac{1}{5}$ $\frac{1}{5}$

b. $3\frac{2}{3} + \frac{2}{3}$

c. $4\frac{4}{6} + \frac{5}{6}$

Lesson 31

Objective: Add mixed numbers.

Homework Key

1. a. 4
   b. $4\frac{4}{5}$
   c. 5
2. a. $4\frac{1}{4}$; number line used
   b. $6\frac{3}{6}$; number line used
   c. $3\frac{4}{12}$; number line used
3. a. $4\frac{2}{4}$; arrow way used to make one
   b. $6\frac{3}{8}$; arrow way used to make one
   c. $6\frac{3}{9}$; arrow way used to make one
4. a. $3\frac{2}{5}$
   b. $5\frac{3}{10}$
   c. $6\frac{4}{7}$

Homework Samples

1. Solve.

a. $2\frac{1}{3} + 1\frac{2}{3} = 3 + \frac{3}{3} = 4$

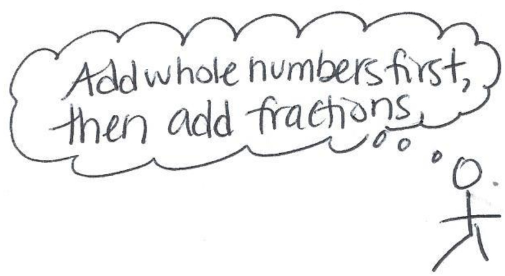
$2 \quad \frac{1}{3} \quad 1 \quad \frac{2}{3}$

b. $2\frac{2}{5} + 2\frac{2}{5} = 4 + \frac{4}{5} = 4\frac{4}{5}$

$2 \quad \frac{2}{5} \quad 2 \quad \frac{2}{5}$

c. $3\frac{3}{8} + 1\frac{5}{8} = 4 + \frac{8}{8} = 4 + 1 = 5$

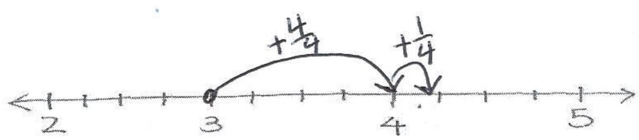
$3 \quad \frac{3}{8} \quad 1 \quad \frac{5}{8}$



2. Solve. Use a number line to show your work.

a. $2\frac{2}{4} + 1\frac{3}{4} = 3 + \frac{5}{4} = \underline{4\frac{1}{4}}$

$\frac{4}{4} \quad \frac{1}{4}$



3. Solve. Use the arrow way to show how to make one.

a. $2\frac{3}{4} + 1\frac{3}{4} = 3\frac{3}{4} + \frac{3}{4} = 4\frac{2}{4}$

$\frac{1}{4} \quad \frac{2}{4}$

$3\frac{3}{4} \xrightarrow{+\frac{1}{4}} 4 \xrightarrow{+\frac{2}{4}} 4\frac{2}{4}$

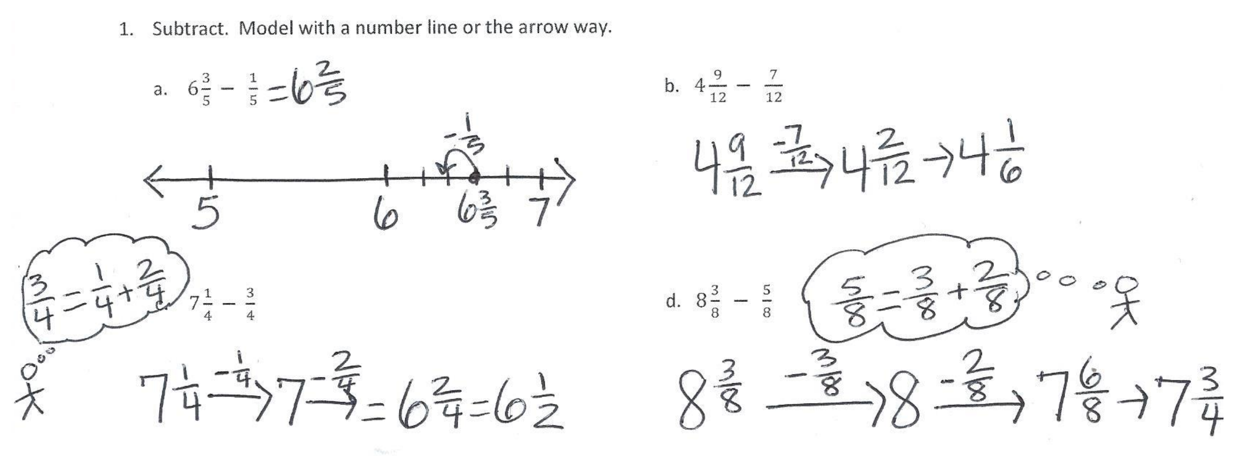
Lesson 32

Objective: Subtract a fraction from a mixed number.

Homework Key

1. a. $6\frac{2}{5}$; number line or arrow way drawn
   b. $4\frac{2}{12}$; number line or arrow way drawn
   c. $6\frac{2}{4}$; number line or arrow way drawn
   d. $7\frac{6}{8}$; number line or arrow way drawn
2. a. $1\frac{3}{5}$; number line or arrow way drawn
   b. $1\frac{2}{3}$; number line or arrow way drawn
   c. $3\frac{3}{6}$; number line or arrow way drawn
   d. $2\frac{4}{6}$; number line or arrow way drawn
   e. $8\frac{4}{8}$; number line or arrow way drawn
   f. $6\frac{5}{10}$; number line or arrow way drawn
   g. $9\frac{4}{8}$; number line or arrow way drawn
   h. $8\frac{9}{12}$; number line or arrow way drawn
   i. $10\frac{4}{5}$; number line or arrow way drawn
   j. $16\frac{5}{9}$; number line or arrow way drawn
3. a. Answer provided
   b. $4\frac{4}{5}$; total decomposed as $4\frac{2}{5}$ and 1
   c. $6\frac{6}{8}$; total decomposed as $6\frac{1}{8}$ and 1
   d. $2\frac{8}{9}$; total decomposed as $2\frac{3}{9}$ and 1
   e. $5\frac{6}{10}$; total decomposed as $5\frac{3}{10}$ and 1
   f. $1\frac{6}{9}$; total decomposed as $1\frac{5}{9}$ and 1

Homework Samples


1. Subtract. Model with a number line or the arrow way.

a. $6\frac{3}{5} - \frac{1}{5} = 6\frac{2}{5}$

b. $4\frac{9}{12} - \frac{7}{12}$

$4\frac{9}{12} \xrightarrow{-\frac{7}{12}} 4\frac{2}{12} \rightarrow 4\frac{1}{6}$

c. $7\frac{1}{4} - \frac{3}{4}$

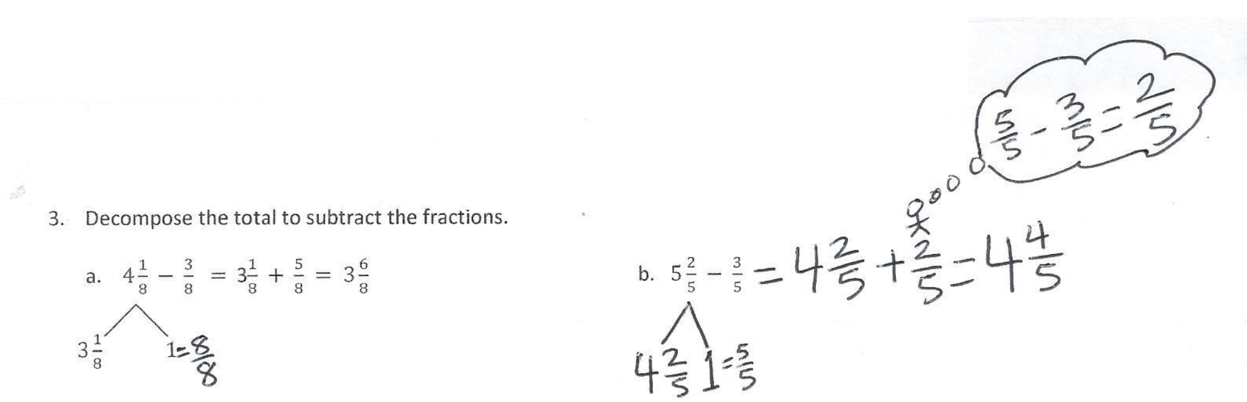
$\frac{3}{4} = \frac{1}{4} + \frac{2}{4}$

$7\frac{1}{4} \xrightarrow{-\frac{1}{4}} 7 \xrightarrow{-\frac{2}{4}} = 6\frac{2}{4} = 6\frac{1}{2}$

d. $8\frac{3}{8} - \frac{5}{8}$

$\frac{5}{8} = \frac{3}{8} + \frac{2}{8}$

$8\frac{3}{8} \xrightarrow{-\frac{3}{8}} 8 \xrightarrow{-\frac{2}{8}} 7\frac{6}{8} \rightarrow 7\frac{3}{4}$


3. Decompose the total to subtract the fractions.

a. $4\frac{1}{8} - \frac{3}{8} = 3\frac{1}{8} + \frac{5}{8} = 3\frac{6}{8}$

$3\frac{1}{8}$ $1 = \frac{8}{8}$

b. $5\frac{2}{5} - \frac{3}{5} = 4\frac{2}{5} + \frac{2}{5} = 4\frac{4}{5}$

$\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$

$4\frac{2}{5}$ $1 = \frac{5}{5}$

Lesson 33

Objective: Subtract a mixed number from a mixed number.

Homework Key

1. a. $1\frac{3}{5}$, $1\frac{3}{5}$
   b. $2\frac{6}{8}$, $2\frac{6}{8} + 2\frac{5}{8} = 5\frac{3}{8}$ or $2\frac{5}{8} + 2\frac{6}{8} = 5\frac{3}{8}$
2. a. Answer provided
   b. $1\frac{4}{7}$; $\frac{4}{7}$ decomposed as $\frac{1}{7}$ and $\frac{3}{7}$
   c. $1\frac{9}{12}$; $\frac{8}{12}$ decomposed as $\frac{5}{12}$ and $\frac{3}{12}$
3. a. $2\frac{6}{8}$
   b. $\frac{7}{12}$; $1\frac{3}{12}$ decomposed as $\frac{3}{12}$ and 1
   c. $2\frac{2}{10}$; $3\frac{1}{10}$ decomposed as $2\frac{1}{10}$ and 1
4. a. $1\frac{7}{9}$
   b. $1\frac{7}{10}$
   c. $2\frac{10}{12}$
   d. $4\frac{12}{100}$

Homework Samples

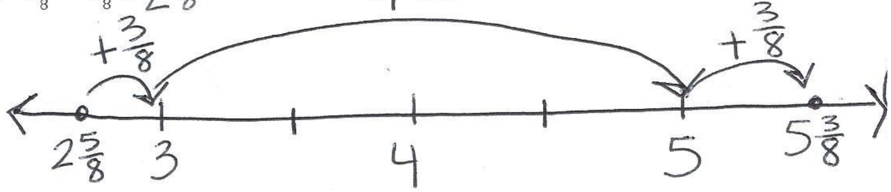
1. Write a related addition sentence. Subtract by counting on. Use a number line or the arrow way to help. The first one has been partially done for you.

   a. $3\frac{2}{5} - 1\frac{4}{5} = 1\frac{3}{5}$

   $1\frac{4}{5} + \underline{1\frac{3}{5}} = 3\frac{2}{5}$

   $$1\frac{4}{5} \xrightarrow{+\frac{1}{5}} 2 \xrightarrow{+1\frac{2}{5}} 3\frac{2}{5}$$

   b. $5\frac{3}{8} - 2\frac{5}{8} = 2\frac{6}{8}$



2. Subtract, as shown in 3(a) below, by decomposing to take one out.

   a. $5\frac{5}{8} - 2\frac{7}{8} = \underbrace{3\frac{5}{8}}_{2\frac{5}{8}\quad 1} - \frac{7}{8} = 2\frac{5}{8} + \frac{1}{8} = 2\frac{6}{8} = 2\frac{3}{4}$

Lesson 34

Objective: Subtract mixed numbers.

Homework Key

1. a. $4\frac{2}{4}$
   b. $5\frac{5}{8}$
   c. $6\frac{5}{6}$
2. a. Answer provided
   b. $1\frac{4}{6}$
   c. $5\frac{6}{8}$
   d. $4\frac{6}{10}$
3. a. $2\frac{6}{12}$
   b. $3\frac{8}{10}$
   c. $7\frac{11}{16}$
   d. $3\frac{11}{100}$

Homework Sample

1. Subtract.

   a. $5\frac{1}{4} - \frac{3}{4}$ $= 4\frac{2}{4}$

   $4 \quad \frac{5}{4}$

   b. $6\frac{3}{8} - \frac{6}{8}$ $= 5\frac{5}{8}$

   $5 \quad \frac{11}{8}$

Grade 4 Module 5 Topic G

Repeated Addition of Fractions as Multiplication

Focus Standard:

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Instructional Days Recommended: 6

Topic G extends the concept of representing repeated addition as multiplication, applying this familiar concept to work with fractions.

Multiplying a whole number times a fraction was introduced in Topic A as students learned to decompose fractions, e.g., $\frac{3}{5} = \frac{1}{5}+\frac{1}{5}+\frac{1}{5} = 3 \times \frac{1}{5}$. In Lessons 35 and 36, students use the associative property, as exemplified below, to multiply a whole number times a mixed number.

3 bananas + 3 bananas + 3 bananas + 3 bananas

= 4 × 3 bananas

= 4 × (3 × 1 banana) = (4 × 3) × 1 banana = 12 bananas

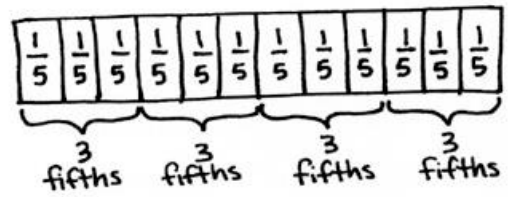
3 fifths + 3 fifths + 3 fifths + 3 fifths

= 4 × 3 fifths

= 4 × (3 fifths) = (4 × 3) fifths= 12 fifths

$$4 \times \frac{3}{5}$$

$$4 \times (3 \text{ x } \frac{1}{5}) = (4 \times 3) \times \frac{1}{5} = \frac{4 \text{ x } 3}{5} = \frac{12}{5}$$



4 × (3 fifths) = (4 × 3) fifths

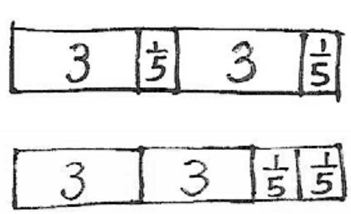
= 12 fifths

4 × 3 fifths = 12 fifths

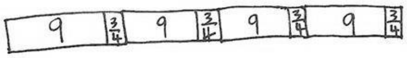
$4 \times \frac{3}{5} = \frac{12}{5}$

Students may have never before considered that 3 bananas = 3 × 1 banana, but it is an understanding that connects place value, whole number work, measurement conversions, and fractions, e.g., 3 hundreds = 3 × 1 hundred or 3 feet = 3 × (1 foot); 1 foot = 12 inches; therefore, 3 feet = 3 × (12 inches) = (3 × 12) inches = 36 inches.

Students explore the use of the distributive property in Lessons 37 and 38 to multiply a whole number by a mixed number. They see the multiplication of each part of a mixed number by the whole number and use the appropriate strategies to do so. As students progress through each lesson, they are encouraged to record only as much as they need to keep track of the math. As shown below, there are multiple steps when using the distributive property, and students can become lost in those steps. Efficiency in solving is encouraged.



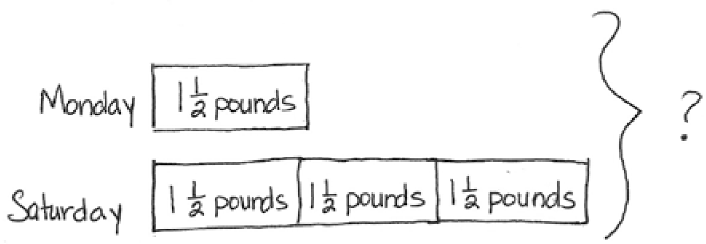
$$2 \times 3\frac{1}{5} = (2 \times 3) + (2 \times \frac{1}{5})$$
$$= 6 + \frac{2}{5} = 6\frac{2}{5}$$



$$4 \times 9\frac{3}{4} = 36 + \frac{12}{4}$$
$$= 36 + 3$$
$$= 39$$

$$5 \times 3\frac{3}{4} = 5 \times (3+\frac{3}{4}) = (5 \times 3) + (5 \times \frac{3}{4}) = 15 + \frac{5 \times 3}{4} = 15 + \frac{15}{4} = 15 + 3\frac{3}{4} = 18\frac{3}{4}$$

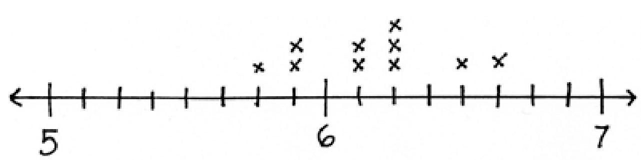
In Lesson 39, students build their problem-solving skills by solving multiplicative comparison word problems involving mixed numbers, e.g., “Jennifer bought 3 times as much meat on Saturday as she did on Monday. If she bought $1\frac{1}{2}$ pounds on Monday, what is the total amount of meat bought for the two days?” They create and use tape diagrams to represent these problems before using various strategies to solve them numerically.



$$4 \times 1\frac{1}{2} = (4 \times 1) + (4 \times \frac{1}{2}) = 4 + \frac{4 \times 1}{2} = 4 + \frac{4}{2} = 4 + 2 = 6$$

Jennifer bought 6 pounds of meat.

In Lesson 40, students solve word problems involving multiplication of a fraction by a whole number. Additionally, students work with data presented in line plots.



**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 35 - 36

Objective: Represent the multiplication of *n* times *a*/*b* as (*n* × *a*)/*b* using the associative property and visual models.

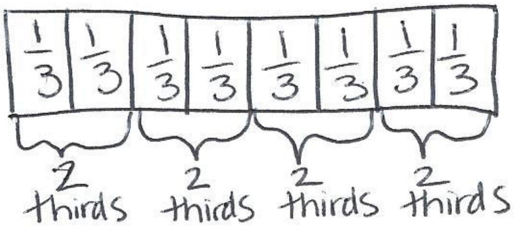
Homework Key (35)

1. a. Tape diagram drawn and labeled
   b. Tape diagram drawn and labeled
2. a. 10 × 2 fifths = 20 fifths
   b. 3 × 5 sixths = 15 sixths
   c. 9 × 4 ninths = 36 ninths
   d. 7 × 3 fourths = 21 fourths
3. a. $\frac{18}{4}$
   b. $\frac{35}{8}$
   c. $\frac{26}{3}$
   d. $\frac{36}{3}$
   e. $\frac{98}{10}$
   f. $\frac{98}{100}$
4. $\frac{10}{3}$ c

Homework Samples

1. Draw and label a tape diagram to show the following are true.

a. 8 thirds = 4 × (2 thirds) = (4 × 2) thirds



2. Write the expression in unit form to solve.

a. $10 \times \frac{2}{5} = \frac{20}{5}$

10 x 2 fifths = 20 fifths

Lesson 36

Homework Key

1. Tape diagram drawn; $4 \times \frac{2}{3}$
2. Tape diagram drawn; $3 \times \frac{7}{8}$
3. a. Answer provided
   b. $3 \times \frac{7}{10} = \frac{21}{10} = 2\frac{1}{10}$
   c. $6 \times \frac{5}{12} = \frac{30}{12} = 2\frac{6}{12}$
   d. $12 \times \frac{3}{8} = \frac{36}{8} = 4\frac{4}{8}$
4. a. $1\frac{5}{9}$
   b. $7\frac{1}{3}$
   c. $13\frac{2}{6}$
   d. 20
   e. $13\frac{4}{5}$
   f. $8\frac{4}{8}$
5. $12\frac{3}{4}$ in
6. $4\frac{1}{8}$
7. 9 ft

Homework Sample

1. Draw a tape diagram to represent

$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$.

$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

Write a multiplication expression equal to

$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$.

$4 \times \frac{2}{3}$

Lesson 37 - 38

Objective: Find the product of a whole number and a mixed number using the distributive property.

Homework Key (37)

1. Two tape diagrams drawn; $3 \times 5\frac{1}{12}$, $(3 \times 5) + \left(3 \times \frac{1}{12}\right)$
2. a. Answer provided

   b. $20\frac{5}{6}$

   c. $15\frac{3}{5}$

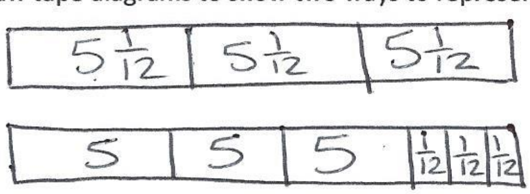
   d. $14\frac{6}{10}$

   e. 58

   f. $40\frac{4}{8}$
3. $13\frac{8}{10}$ mi
4. $28\frac{2}{10}$ lb

Homework Sample

1. Draw tape diagrams to show two ways to represent 3 units of $5\frac{1}{12}$.



Write a multiplication expression to match each tape diagram.

$3 \times 5\frac{1}{12}$ $(3 \times 5) + \left(3 \times \frac{1}{12}\right)$

Lesson 38

Homework Key

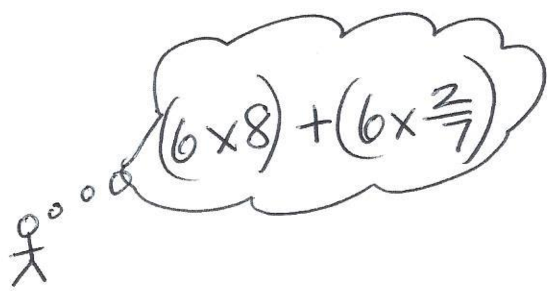
1. a. 8, 8
   b. $7, \frac{7}{10}$
2. a. $49\frac{5}{7}$
   b. $69\frac{3}{4}$
   c. 79
   d. $77\frac{5}{8}$
   e. $82\frac{8}{12}$
   f. $360\frac{36}{100}$
3. $41\frac{5}{8}$ ft
4. $45\frac{2}{4}$ c
5. $215\frac{5}{8}$ oz

Homework Samples

1. Fill in the unknown factors.

   a. $8 \times 4\frac{4}{7} = (\underline{8} \times 4) + (\underline{8} \times \frac{4}{7})$

   b. $9 \times 7\frac{7}{10} = (9 \times \underline{7}) + (9 \times \underline{\frac{7}{10}})$



2. Multiply. Use the distributive property.

   a. $6 \times 8\frac{2}{7}$

$$6\times8\frac{2}{7} = 48+\frac{12}{7}$$
$$=48+1\frac{5}{7}$$
$$=49\frac{5}{7}$$

Lesson 39

Objective: Solve multiplicative comparison word problems involving fractions.

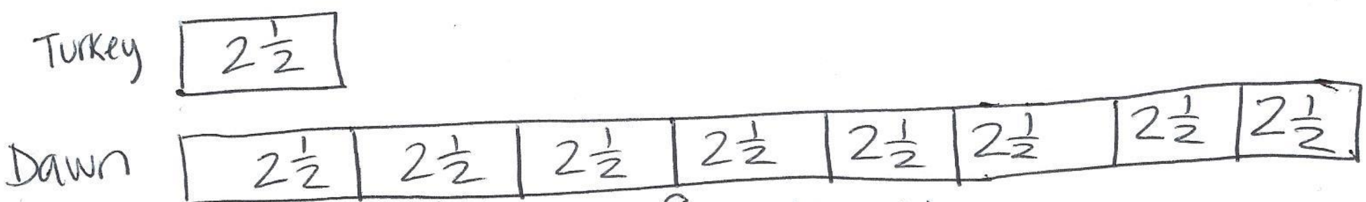
Homework Key

1. 20 lb
2. $15\frac{6}{8}$ in
3. $24\frac{1}{4}$ yd
4. $50\frac{4}{8}$ c
5. $290\frac{8}{10}$ mi
6. $147

Homework Samples

Use the RDW process to solve.

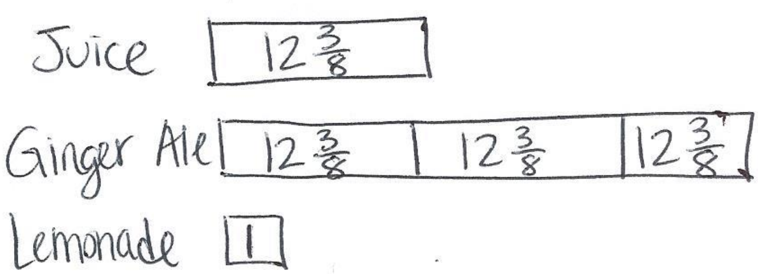
1. Ground turkey is sold in packages of $2\frac{1}{2}$ pounds. Dawn bought eight times as much turkey that is sold in 1 package for her son's birthday party. How many pounds of ground turkey did Dawn buy?



$8 \times 2\frac{1}{2} = 16 + \frac{8}{2} = 16 + 4 = 20$

Dawn bought 20 Lbs of ground turkey.

4. Carol made punch. She used $12\frac{3}{8}$ cups of juice and then added three times as much ginger ale. Then, she added 1 cup of lemonade. How many cups of punch did her recipe make?



$12\frac{3}{8} \times 4 = 48 + \frac{12}{8}$
$= 48 + 1\frac{4}{8}$
$= 48 + 1\frac{1}{2}$
$= 49\frac{1}{2}$
$= 49\frac{1}{2} + 1 = 50\frac{1}{2}$ (→Lemonade)

Carol made $50\frac{1}{2}$ cups of punch.

Lesson 40

Objective: Solve word problems involving the multiplication of a whole number and a fraction including those involving line plots.

Homework Key

1. Line plot drawn accurately
2. $2\frac{7}{8}$ in
3. $1\frac{5}{8}$ in
4. $9\frac{3}{8}$ in
5. $1\frac{1}{8}$ in
6. August and October
7. $22\frac{4}{8}$ in

Homework Sample

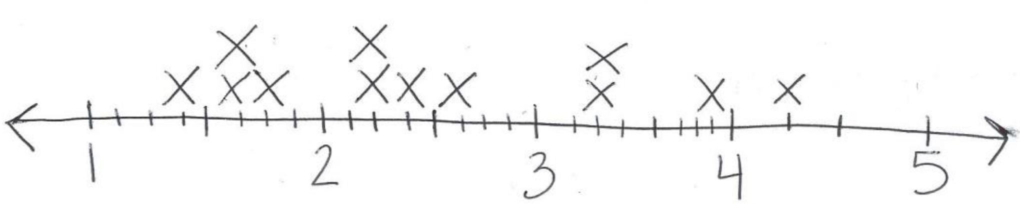
The chart to the right shows the total monthly rainfall for a city.

1. Use the data to create a line plot at the bottom of this page and to answer the following questions.

Month	Rainfall (in inches)
January	$2\frac{2}{8}$
February	$1\frac{3}{8}$
March	$2\frac{3}{8}$
April	$2\frac{5}{8}$
May	$4\frac{1}{4}$
June	$2\frac{1}{4}$
July	$3\frac{7}{8}$
August	$3\frac{1}{4}$
September	$1\frac{5}{8}$
October	$3\frac{2}{8}$
November	$1\frac{3}{4}$
December	$1\frac{5}{8}$

$\frac{2}{8} = \frac{1}{4}$

$\frac{2}{8} = \frac{1}{4}$



Grade 4 Module 5 Topic H

Exploring a Fraction Pattern

Focus Standard:

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Instructional Days Recommended: 1

Topic H is an exploration lesson in which students find the sum of all like denominators from $\frac{0}{n}$ to $\frac{n}{n}$.

Students first work, in teams, with fourths, sixths, eighths, and tenths. For example, they might find the sum of all sixths from $\frac{0}{6}$ to $\frac{6}{6}$. Students discover that they can make pairs with a sum of 1 to add more efficiently, e.g., $\frac{0}{6}+\frac{6}{6}$, $\frac{1}{6}+\frac{5}{6}$, $\frac{2}{6}+\frac{4}{6}$, and there will be one fraction, $\frac{3}{6}$, without a pair. As students make this discovery, they share and compare their strategies within their teams. They then extend this to similarly find sums of thirds, fifths, sevenths, and ninths, observing patterns when finding the sum of odd and even denominators (**4.OA.5**). Through discussion of their strategies, students determine which are most efficient.

Advanced students can be challenged to find the sum of all hundredths from 0 hundredths to 100 hundredths.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 41

Objective: Find and use a pattern to calculate the sum of all fractional parts between 0 and 1. Share and critique peer strategies.

Homework Key

1. a. 3
   b. $3\frac{1}{2}$
   c. 4
   d. $4\frac{1}{2}$
   e. 5
   f. $5\frac{1}{2}$
2. Answers will vary.
3. The sums would remain the same.
4. a. $10\frac{10}{20}$
   b. 18
   c. $18\frac{18}{36}$
   d. 38
   e. $50\frac{50}{100}$
   f. 50
5. Answers will vary.

Homework Sample

1. Find the sums.

a. $\frac{0}{5}+\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}+\frac{5}{5}$

$\left(\frac{0}{5}+\frac{5}{5}\right)+\left(\frac{1}{5}+\frac{4}{5}\right)+\left(\frac{2}{5}+\frac{3}{5}\right)=3$

b. $\frac{0}{6}+\frac{1}{6}+\frac{2}{6}+\frac{3}{6}+\frac{4}{6}+\frac{5}{6}+\frac{6}{6}$

$\left(\frac{0}{6}+\frac{6}{6}\right)+\left(\frac{1}{6}+\frac{5}{6}\right)+\left(\frac{2}{6}+\frac{4}{6}\right)+\frac{3}{6}=3\frac{3}{6}=3\frac{1}{2}$